`sftfe`: A Stata command for fixed-effects stochastic frontier models estimation

Federico Belotti[⋆] Giuseppe Ilardi[∘]

[⋆]CEIS, University of Rome Tor Vergata
[∘]Bank of Italy

2014 Italian Stata Users Group Meeting
Milan. November 13, 2014

Outline

1. Contribution to the community
2. Consistent estimation of the fixed-effects SF model
3. The `sftfe` command
4. Monte Carlo results

The fixed-effects stochastic frontier (SF) model

$$y_{it} = \alpha_i + \mathbf{x}_{it}\boldsymbol{\beta} + \varepsilon_{it}, \quad (1)$$

$$\varepsilon_{it} = v_{it} - u_{it}, \quad i = 1, \ldots, n, \quad t = 1, \ldots, T, \quad (2)$$

where, for each unit i and period t:

- y_{it} represents the output;
- $\mathbf{x}_{it}$ is a $1 \times k$ vector of exogenous inputs;
- $\boldsymbol{\beta}$ is a $k \times 1$ vector of technology parameters;
- α_i is the unit fixed-effect;
- v_{it} is the idiosyncratic error;
- u_{it} the one-sided disturbance which represents inefficiency.

Distributional assumptions - homoskedastic model

$$v_{it} \sim IID \ \mathcal{N}(0, \psi^2), \tag{3}$$

$$u_{it} \sim IID \ \mathcal{F}_u(\mu, \sigma^2), \quad i = 1, \ldots, n, \quad t = 1, \ldots, T, \tag{4}$$

- v_{it} and u_{it} are independently distributed;
- The inefficiency u_{it} has distribution with support defined over $\mathbb{R}^+$, mean μ and variance parameter σ^2 (e.g., half-normal ($\mu = 0$), exponential ($\mu = \sigma$) or truncated-normal);
- v_{it} is normally distributed with variance ψ^2.

Heterogeneity

- **Heterogeneity**: can be *observable* or *unobservable*;
- Model (1)-(2) adds α_i (unobservable) to shift the production (cost) function;
- Observable heterogeneity is reflected in measured variables;
- Examples are:
  1. Heteroskedastic inefficiency $\rightarrow \sigma_{it} = \exp(\mathbf{z}_{it}\boldsymbol{\delta})$;
  2. Heteroskedastic noise $\rightarrow \psi_{it} = \exp(\mathbf{r}_{it}\boldsymbol{\gamma})$;
  3. Heterogeneity in the inefficiency mean $\rightarrow \mu_{it} = \mathbf{s}_{it}\boldsymbol{\xi}$;
- It might be that $\mathbf{z}_{it} = \mathbf{r}_{it} = \mathbf{s}_{it}$.

The Maximum Dummy Variable approach

- Greene (2005) propose to estimate model (1)−(4) by treating the unit-specific intercepts as parameters to be estimated;
- This approach has been implemented in the `sfpanel` command (Belotti et al., 2013);
- However, as Greene's simulations suggest, this approach leads to inconsistent variance estimates, especially in short panels.
- Since these parameters represent the key ingredients in the post-estimation of inefficiencies, a solution to this issue is crucial in the SF context.

Our contribution

- The new command `sftfe` allows the estimation of the fixed-effects SF models via three alternative estimators (Belotti and Ilardi, 2012; Chen et al., 2014)[1];
- They exploit the first-difference data transformation to eliminate the fixed-effects achieving consistency for both fixed-n and fixed-T asymptotics;
- `sftfe` allows to estimate models in which inefficiency follows a first-order autoregressive process as well as to model inefficiency's variance (eventually also the mean) as a function of exogenous covariates.

[1] Belotti and Ilardi (2012) has been revised including the extension of the Chen et al. (2014) approach to heteroskedastic and dynamic inefficiency models. The updated version is available from `http://www.econometrics.it`.

Eliminate the nuisance parameters

We get rid of the nuisance parameters through a **first-difference** data transformation

$$\begin{aligned} \Delta \mathbf{y}_i &= \Delta X_i \boldsymbol{\beta} + \Delta \boldsymbol{\varepsilon}_i, & (5) \\ \Delta \boldsymbol{\varepsilon}_i &= \Delta \mathbf{v}_i - \Delta \mathbf{u}_i, & (6) \end{aligned}$$

where $\Delta \mathbf{y}_i = (\Delta y_{i2}, \ldots, \Delta y_{iT})$ with $\Delta y_{it} = y_{it} - y_{it-1}$ and ΔX_i is the $T - 1 \times k$ matrix of time-varying covariates with the t-th row denoted by $\Delta \mathbf{x}_{it} = (\Delta x_{it1}, \ldots, \Delta x_{itk})$, $\forall\ t = 2, \ldots, T$.

First-differenced model

Idiosyncratic error - $\Delta \mathbf{v}_i$

The normality assumption for v_{it} implies that $\Delta \mathbf{v}_i$ has a $T-1$-variate normal distribution with covariance matrix $\Psi = \psi^2 \Lambda_{T-1}$, where Λ_{T-1} is

$$\Lambda_{T-1} = \begin{pmatrix} 2 & -1 & 0 & \cdots & 0 \\ -1 & 2 & -1 & \cdots & 0 \\ 0 & -1 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & -1 \\ 0 & 0 & \dots & -1 & 2 \end{pmatrix} \quad (7)$$

First-differenced model (*ctd*)

Inefficiency - $\Delta\mathbf{u}_i$

- The multivariate distribution of $\Delta\mathbf{u}_i$ is **generally unknown**;
- Nevertheless, given the independence assumption between $\Delta\mathbf{v}_i$ and $\Delta\mathbf{u}_i$, the marginal likelihood contribution L_i^* can be defined in general terms as

$$\begin{aligned} L_i^*(\boldsymbol{\theta}) &= \int f(\Delta\mathbf{v}_i, \Delta\mathbf{u}_i)\, d\Delta\mathbf{u}_i = \int f(\Delta\mathbf{v}_i) f(\Delta\mathbf{u}_i)\, d\Delta\mathbf{u}_i \\ &= \int f(\Delta\mathbf{y}_i | \Delta\mathbf{u}_i) f(\Delta\mathbf{u}_i)\, d\Delta\mathbf{u}_i \end{aligned} \tag{8}$$

where $\boldsymbol{\theta}$ is the parameter vector to be estimated.

How to estimate the model: MMLE

- Marginal Maximum Likelihood estimator (MMLE, Chen et al., 2014);

  1. The basic idea is to exploit the Closed Skew Normal class of distributions (CSN, Gonzalez-Farias et al., 2004) that, thanks to its closeness property under marginalization and linear transformations, allows to derive a closed form expression for the **marginal** likelihood function in equation (8);
  2. Feasible only when inefficiency has truncated-normal (or half-normal) distribution;
  3. Extension to heteroskedastic (or dynamic) inefficiency is cumbersome when $T > 5$ since the estimation requires the approximation of T-variate Gaussian integrals (see Kumbhakar and Tsionas, 2011; Chen et al., 2014).

How to estimate the model: MMSLE

- Marginal Maximum Simulated Likelihood estimator (MMSLE, Belotti and Ilardi, 2012);

1. The basic idea is that estimation can be accomplished via simulation, treating the marginal likelihood function in equation (8) as an expectation with respect to the random vector $\mathbf{\Delta u}_i$;
2. Feasible when inefficiency has half-normal or exponential distribution;
3. Extension to heteroskedastic inefficiency is feasible but constrained (only time-invariant covariates can be used to model inefficiency variability);
4. Extension to dynamic inefficiency not feasible.

How to estimate the model: ...

- The MMLE is cumbersome when the inefficiency (and/or the idiosyncratic error) is allowed to be heteroskedastic and $T > 5$;
- The MMSLE imposes a restriction: the variance can only be expressed as a function of **time-invariant** exogenous explanatory variables.
- **Solution**: Pairwise Difference estimator (PDE, Belotti and Ilardi, 2012).

How to estimate the model: PDE

- Pairwise Difference estimator (PDE, Belotti and Ilardi, 2012);

  1. The basic idea is to exploit the closeness property of the normal-exponential (or the normal-truncated normal via the CSN framework) marginal likelihood function when $T = 2$ to define a U-estimator based on all pairwise quasi likelihood contributions;
  2. Feasible and computationally efficient when inefficiency is heteroskedastic and has half-normal, exponential or truncated-normal distribution;
  3. Extension to dynamic inefficiency is feasible and straightforward when the latter has truncated-normal (or half-normal) distribution.

The basic `sftfe` syntax is the following

`sftfe` *depvar* [*indepvars*] [*if*] [*in*] [, options]

Factor variables are allowed.

Options:

estimator(*type*) specifies the estimator to be used. May be *mmle*, *mmsle* and *pde*. Default is *pde*.

cost specifies a cost frontier model; default is production frontier model.

MMLE's specific options

- distribution(*distname*) specifies the inefficiency distribution. Can be *hnormal* or *tnormal*. Default is *hnormal*.

- ghkdraws(*[#]* , [type(*string*) antithetics]) governs the draws used in Geweke-Hajivassiliou-Keane (GHK) simulation of higher-dimensional cumulative multivariate normal distributions. if # is omitted, the number of draws is set to 100. The type(*string*) suboption specifies the type of sequence in the simulation, can be *halton*, *hammersley*, *ghalton*, *random*, with *halton* being the default; antithetics requests antithetic draws; If this option is omitted, the estimation is performed exploiting the result outlined in Kotz et al. (2000) through Gauss-Hermite quadrature.

MMSLE's specific options

- `distribution(`*distname*`)` specifies the inefficiency distribution. Can be *exponential* or *hnormal*. Default is *exponential*.
- `usigma(`*varlist* [, noconstant]`)` specifies that inefficiency is heteroscedastic, with variance expressed as a function of **time-invariant** covariates defined in *varlist*. Specifying `noconstant` suppresses the intercept in this function.

MMSLE's specific options - 2

- `simtype(`*string*`)` specifies the method to generate random draws for the first-differenced inefficiency. Can be *uniform*, for uniformly distributed random variates, or *halton* (the default) for Halton sequences.

- `nsimulations(#)` specifies the number of draws used in the simulation. The default is 250.

- `base(#)` specifies the number, preferably a prime, used as a base for the generation of Halton sequences. The default is 5.

PDE's specific options

- `distribution`(*distname*) specifies the inefficiency distribution. Can be *exponential*, *hnormal* or *tnormal*. Default is *hnormal*.
- `dynamic` specifies that inefficiency follows a first-order autoregressive process. Only when `distribution`(*distname*) is *hnormal* or *tnormal*.

PDE's specific options - 2

- `emean(`*varlist_m* [, noconstant]`)` may be used only with `distribution(tnormal)`. With this option, `sftfe` specifies the inefficiency mean as a linear function of the covariates defined in *varlist_m*.*

- `usigma(`*varlist_u* [, noconstant]`)` specifies that inefficiency is heteroscedastic, with variance expressed as a function of covariates defined in *varlist_u*.*

- `vsigma(`*varlist_v* [, noconstant]`)` specifies that idiosyncratic error is heteroscedastic, with variance expressed as a function of covariates defined in *varlist_v*.*

* Specifying `noconstant` suppresses the constant in this function.

Postestimation

`predict [type] newvar [if] [in] [, statistic]`

where statistic includes:

- xb, the default, calculates the linear prediction.
- stdp calculates the standard error of the linear prediction.
- u produces estimates of (technical or cost) inefficiency via $\mathbb{E}(u|\varepsilon)$ using the Jondrow et al. (1982) estimator.
- jlms produces estimates of (technical or cost) efficiency via $\exp\left[-\mathbb{E}(u|\varepsilon)\right]$.
- alpha produces estimates of fixed-effects.

Syntax examples

Homoskedastic normal-truncated normal model via MMLE:

```
sftfe y x1 x2, est(mmle) dist(tn)
```

Homoskedastic normal-exponential model via MMLE:

```
sftfe y x1 x2, est(mmsle) dist(exp) nsim(250) base(7)
```

Heteroskedastic normal-exponential model via PDE:

```
sftfe y x1 x2, est(pde) dist(exp) usigma(z1 z2)
```

Heteroskedastic and dynamic normal-half normal model via PDE:

```
sftfe y x1 x2, est(pde) dist(hn) dynamic usigma(z1 z2)
```

MMSLE vs MMLE - Data Generating Process

We consider the homoskedastic normal-half normal model investigated by Chen et al. (2014), that is

$$y_{it} = \alpha_i + \beta x_{it} + v_{it} - u_{it}, \tag{9}$$

$$v_{it} \sim \mathcal{N}(0, \psi^2), \tag{10}$$

$$u_{it} \sim \mathcal{N}^+ \left(0, \sigma^2\right) \quad i = 1, \ldots, n, \quad t = 1, \ldots, T, \tag{11}$$

where

- the fixed-effect parameters $\alpha_1, ..., \alpha_n$ are drawn from a standard Gaussian random variable; $x_{it} = 0.5\alpha_i + \sqrt{0.5^2} w_{it}$ with $w_{it} \sim \mathcal{N}(0, 1)$;
- For each experiment, we use the same α_i and x_{it} in all replications, thus only u_{it} and v_{it} are redrawn in each replication;
- We set $\beta = 1$, $\frac{\sigma}{\psi} = \lambda = 2$, and consider different sample sizes ($n = 100, 250$) and panel lengths ($T = 5, 10$);
- The analysis is based on 250 replications for each experiment.

Results: MMSLE vs MMLE ($n = 100$)

$T = 5$

	MMSLE		MMLE	
	Bias	MSE	Bias	MSE
β	-0.002	0.004	-0.002	0.004
σ	-0.025	0.028	-0.050	0.061
ψ	-0.006	0.010	4.5e-04	0.012
$E(u\|\varepsilon)$	-0.036	0.324	-0.055	0.348
$r_{u,\hat{u}}$	0.707		0.707	
	(0.644)		(0.644)	

$T = 10$

	MMSLE		MMLE	
	Bias	MSE	Bias	MSE
β	-0.001	0.001	-0.001	0.001
σ	-0.051	0.010	-0.007	0.009
ψ	0.036	0.003	0.004	0.003
$E(u\|\varepsilon)$	-0.047	0.268	-0.012	0.266
$r_{u,\hat{u}}$	0.752		0.752	
	(0.692)		(0.692)	

Results: MMSLE vs MMLE ($n = 100$)

$T = 5$

	MMSLE		MMLE	
	Bias	MSE	Bias	MSE
β	-0.002	0.004	-0.002	0.004
σ	-0.025	0.028	-0.050	0.061
ψ	-0.006	0.010	4.5e-04	0.012
$E(u\|\varepsilon)$	-0.036	0.324	-0.055	0.348
$r_{u,\hat{u}}$	0.707		0.707	
	(0.644)		(0.644)	

$T = 10$

	MMSLE		MMLE	
	Bias	MSE	Bias	MSE
β	-0.001	0.001	-0.001	0.001
σ	-0.051	0.010	-0.007	0.009
ψ	0.036	0.003	0.004	0.003
$E(u\|\varepsilon)$	-0.047	0.268	-0.012	0.266
$r_{u,\hat{u}}$	0.752		0.752	
	(0.692)		(0.692)	

Results: MMSLE vs MMLE ($n = 100$)

$T = 5$

	MMSLE		MMLE	
	Bias	MSE	Bias	MSE
β	-0.002	0.004	-0.002	0.004
σ	-0.025	0.028	-0.050	0.061
ψ	-0.006	0.010	4.5e-04	0.012
$E(u\|\varepsilon)$	-0.036	0.324	-0.055	0.348
$r_{u,\hat{u}}$	0.707		0.707	
	(0.644)		(0.644)	

$T = 10$

	MMSLE		MMLE	
	Bias	MSE	Bias	MSE
β	-0.001	0.001	-0.001	0.001
σ	-0.051	0.010	-0.007	0.009
ψ	0.036	0.003	0.004	0.003
$E(u\|\varepsilon)$	-0.047	0.268	-0.012	0.266
$r_{u,\hat{u}}$	0.752		0.752	
	(0.692)		(0.692)	

Results: MMSLE vs MMLE ($n = 100$)

$T = 5$

	MMSLE		MMLE	
	Bias	MSE	Bias	MSE
β	-0.002	0.004	-0.002	0.004
σ	-0.025	0.028	-0.050	0.061
ψ	-0.006	0.010	4.5e-04	0.012
$E(u\|\varepsilon)$	-0.036	0.324	-0.055	0.348
$r_{u,\hat{u}}$	0.707		0.707	
	(0.644)		(0.644)	

$T = 10$

	MMSLE		MMLE	
	Bias	MSE	Bias	MSE
β	-0.001	0.001	-0.001	0.001
σ	-0.051	0.010	-0.007	0.009
ψ	0.036	0.003	0.004	0.003
$E(u\|\varepsilon)$	-0.047	0.268	-0.012	0.266
$r_{u,\hat{u}}$	0.752		0.752	
	(0.692)		(0.692)	

Results: MMSLE vs MMLE ($n = 100$)

$T = 5$

	MMSLE		MMLE	
	Bias	MSE	Bias	MSE
β	-0.002	0.004	-0.002	0.004
σ	-0.025	0.028	-0.050	0.061
ψ	-0.006	0.010	4.5e-04	0.012
$E(u\|\varepsilon)$	-0.036	0.324	-0.055	0.348
$r_{u,\hat{u}}$	0.707		0.707	
	(0.644)		(0.644)	

$T = 10$

	MMSLE		MMLE	
	Bias	MSE	Bias	MSE
β	-0.001	0.001	-0.001	0.001
σ	-0.051	0.010	-0.007	0.009
ψ	0.036	0.003	0.004	0.003
$E(u\|\varepsilon)$	-0.047	0.268	-0.012	0.266
$r_{u,\hat{u}}$	0.752		0.752	
	(0.692)		(0.692)	

The bias may be reduced by increasing the number of draws

Results: MMSLE vs MMLE ($n = 100$)

$T = 5$

	MMSLE		MMLE	
	Bias	MSE	Bias	MSE
β	-0.002	0.004	-0.002	0.004
σ	-0.025	0.028	-0.050	0.061
ψ	-0.006	0.010	4.5e-04	0.012
$E(u\|\varepsilon)$	-0.036	0.324	-0.055	0.348
$r_{u,\hat{u}}$	0.707		0.707	
	(0.644)		(0.644)	

$T = 10$

	MMSLE		MMLE	
	Bias	MSE	Bias	MSE
β	-0.001	0.001	-0.001	0.001
σ	-0.051	0.010	-0.007	0.009
ψ	0.036	0.003	0.004	0.003
$E(u\|\varepsilon)$	-0.047	0.268	-0.012	0.266
$r_{u,\hat{u}}$	0.752		0.752	
	(0.692)		(0.692)	

Results: MMSLE vs MMLE ($n = 250$)

$T = 5$

	MMSLE		MMLE	
	Bias	MSE	Bias	MSE
β	0.001	0.001	0.001	0.001
σ	0.002	0.011	0.001	0.012
ψ	-0.011	0.004	-0.011	0.005
E(u$\vert\varepsilon$)	-0.016	0.304	-0.017	0.305
$r_{u,\hat{u}}$	0.711		0.711	
	(0.651)		(0.651)	

$T = 10$

	MMSLE		MMLE	
	Bias	MSE	Bias	MSE
β	0.001	3.7e-04	8.8e-04	3.6e-04
σ	-0.025	0.004	-0.004	0.004
ψ	0.016	0.001	-2.9e-04	0.001
E(u$\vert\varepsilon$)	-0.026	0.261	-0.009	0.261
$r_{u,\hat{u}}$	0.752		0.752	
	(0.691)		(0.691)	

Dynamic PDE - Data Generating Process

We specify the following heteroskedastic normal-half normal model with $AR(1)$ inefficiencies

$$\begin{aligned} \mathbf{y}_i &= \alpha_i \iota_T + \beta \mathbf{x}_i + \mathbf{v}_i - \mathbf{u}_i, & (12) \\ \mathbf{v}_i &\sim \mathcal{N}_T(0, \psi^2 I_t), & (13) \\ \mathbf{u}_i &\sim \mathcal{N}_T^+\left(\mathbf{0}, (1-\rho^2)^{-1}\Omega_i\right), \quad i = 1, \dots, n, & (14) \end{aligned}$$

where

- $\Omega_i = \{\omega_{its}\}^{t,s=1,\dots,T}$ with $\omega_{its} = \sigma_{it}\sigma_{is}\rho^{|t-s|}$ and $\sigma_{it} = \exp(\gamma_0 + z_{it}\gamma_1)$;
- $\alpha_1, \dots, \alpha_n$ and z_{it} are drawn from a standard Gaussian random variable while $x_{it} = 0.5\alpha_i + \sqrt{0.5^2} w_{it}$ with $w_{it} \sim \mathcal{N}(0, 1)$;
- We set $\beta = 0.5$, $\psi = 0.5$, $\gamma_0 = -0.5$ and $\gamma_1 = 1$ (this implies $\bar{\lambda} = \frac{1}{nT\psi}\sum_{i=1}^{n}\sum_{t=1}^{T}\sigma_{it} \approx 2$).

Dynamic PDE - Data Generating Process

- The simulation of the inefficiency vector $\mathbf{u}_i$ is performed using the MCMC approach outlined in Geweke (1991), which uses a Gibbs algorithm for sampling from an arbitrary multivariate truncated normal distribution;

- We consider two different values for the ρ parameter ($\rho = 0.3, 0.7$), different sample sizes ($n = 100, 250$) and panel lengths ($T = 5, 10$);

- The analysis is based on 250 replications for each experiment.

Dynamic PDE ($\rho = 0.3$, $n = 100$)

$T = 5$

	Bias	MSE
β	-0.002	0.001
γ_0	-0.061	0.051
γ_1	-0.013	0.011
ψ	-0.002	0.001
ρ	0.071	0.039
$E(u \mid \varepsilon)$	0.028	0.249
$r_{u,\hat{u}}$	0.952	
	(0.781)	

$T = 10$

	Bias	MSE
β	-7.8e-04	5.5e-04
γ_0	-0.009	0.019
γ_1	-0.004	0.005
ψ	5.4e-04	6.0e-04
ρ	0.034	0.024
$E(u \mid \varepsilon)$	0.032	0.173
$r_{u,\hat{u}}$	0.970	
	(0.806)	

Dynamic PDE ($\rho = 0.3$, $n = 250$)

$T = 5$

	Bias	MSE
β	-0.001	4.3e-04
γ_0	-0.013	0.016
γ_1	-0.009	0.004
ψ	-0.001	6.2e-04
ρ	0.018	0.023
E(u|ε)	0.018	0.233
$r_{u,\hat{u}}$	0.957	
	(0.784)	

$T = 10$

	Bias	MSE
β	-6.9e-04	2.1e-04
γ_0	0.009	0.007
γ_1	-0.010	0.002
ψ	0.002	2.5e-04
ρ	0.036	0.011
E(u|ε)	0.037	0.170
$r_{u,\hat{u}}$	0.971	
	(0.810)	

Dynamic PDE ($\rho = 0.7$, $n = 100$)

$T = 5$

	Bias	MSE
β	-0.003	0.001
γ_0	-0.220	0.104
γ_1	-0.012	0.007
ψ	-6.3e-04	0.002
ρ	-0.008	0.010
E(u|ε)	-0.123	0.515
$r_{u,\hat{u}}$	0.955	
	(0.800)	

$T = 10$

	Bias	MSE
β	-9.7e-04	7.2e-04
γ_0	-0.088	0.030
γ_1	-0.010	0.003
ψ	0.002	7.1e-04
ρ	-0.019	0.004
E(u|ε)	-0.050	0.346
$r_{u,\hat{u}}$	0.974	
	(0.838)	

Dynamic PDE ($\rho = 0.7$, $n = 250$)

$T = 5$

	Bias	MSE
β	-0.001	5.2e-04
γ_0	-0.180	0.054
γ_1	-0.012	0.003
ψ	-0.002	7.1e-04
ρ	-0.011	0.004
E(u|ε)	-0.106	0.496
$r_{u,\hat{u}}$	0.959	
	(0.801)	

$T = 10$

	Bias	MSE
β	-3.9e-04	2.5e-04
γ_0	-0.073	0.012
γ_1	-0.013	0.001
ψ	0.002	3.0e-04
ρ	-0.019	0.002
E(u|ε)	-0.039	0.336
$r_{u,\hat{u}}$	0.975	
	(0.842)	

References

Belotti, F., Daidone, S., Ilardi, G., and Atella, V. (2013). Stochastic frontier analysis using Stata. *Stata Journal*, 13(4):718–758.

Belotti, F. and Ilardi, G. (2012). CEIS Research Paper 231, Tor Vergata University, CEIS.

Chen, Y., Wang, H., and Schmidt, P. (2014). Consistent estimation of the fixed effects stochastic frontier model. *Journal of Econometrics*, 181(2):65–76.

Geweke, J. (1991). Efficient simulation from the multivariate normal and student-t distributions subject to linear constraints and the evaluation of constraint probabilities. In Keramidas, E. M., editor, *Computing Science and Statistics: Proceedings of the 23rd Symposium on the Interface*, pages 571–578. Interface Foundation of North America, Inc.

Gonzalez-Farias, G., Dominguez-Molina, J., and Gupta, A. (2004). Additive properties of skew normal random vectors. *Journal of Statistical Planning and Inference*, 126:521–534.

Greene, W. (2005). Reconsidering heterogeneity in panel data estimators of the stochastic frontier model. *Journal of Econometrics*, 126:269–303.

Jondrow, J., Lovell, C., Materov, I., and Schmidt, P. (1982). On the estimation of technical efficiency in the stochastic production function model. *Journal of Econometrics*, 19:233–238.

Kotz, S., Balakrishnan, N., and Johnson, N. L. (2000). *Continuous Multivariate Distributions, Volume 1, Models and Applications,* 2^{nd} *Edition*. John Wiley & Sons.

Kumbhakar, S. C. and Tsionas, E. G. (2011). Some recent developments in efficiency measurement in stochastic frontier models. *Journal of Probability and Statistics*, 2011.