Nonlinear dynamic stochastic general equilibrium models in Stata 16

David Schenck

Senior Econometrician
Stata

2019 Italian Stata User Group Meeting
September 26, 2019

Motivation

- Models used in macroeconomics for policy analysis
- Models for multiple time series
- linking observed variables to latent factors
- and the link is motivated by economic theory
- Alternatively: methods for bringing theoretical macroeconomic models to the data

Here's a model

- Households demand output, given inflation and interest rates:

$$\frac{1}{X_t} = \beta E_t \left[\left(\frac{1}{X_{t+1}} \right) \left(\frac{R_t}{\Pi_{t+1} Z_{t+1}} \right) \right]$$

- Firms set prices, given output demand:

$$\phi + (\Pi_t - 1) = \frac{1}{\phi} X_t + \beta E_t \left[\Pi_{t+1} - 1 \right]$$

- Central bank sets interest rate, given inflation

$$\beta R_t = \Pi_t^{1/\beta} M_t$$

Here's a model

- The model's control variables are determined by equations:

$$\frac{1}{X_t} = \beta E_t \left[\left(\frac{1}{X_{t+1}} \right) \left(\frac{R_t}{\Pi_{t+1} Z_{t+1}} \right) \right]$$

$$\phi + (\Pi_t - 1) = \frac{1}{\phi} X_t + \beta E_t \left[\Pi_{t+1} - 1 \right]$$

$$\beta R_t = \Pi_t^{1/\beta} M_t$$

- The model is completed by adding equations for the state variables:

$$\ln(Z_{t+1}) = \rho_z \ln(Z_t) + \xi_{t+1}$$

$$\ln(M_{t+1}) = \rho_m \ln(M_t) + e_{t+1}$$

Here's a model in Stata

```
. dsgenl  (1 = {beta}*(F.x/x)^(-1)*(r/(F.p*F.z)))        ///
          ({phi}+(p-1) = 1/{phi}*x + {beta}*(F.p-1))   ///
          ({beta}*r = p^(1/{beta})*m)                  ///
          (ln(F.m) = {rhom}*ln(m))                     ///
          (ln(F.z) = {rhoz}*ln(z))                     ///
          , exostate(z m) observed(p r) unobserved(x)
```

Parameter estimation

```
. dsgenl  (1 = {beta}*(F.x/x)^(-1)*(r/(F.p*F.z)))      ///
>         ({phi}+(p-1) = 1/{phi}*x + {beta}*(F.p-1))   ///
>         ({beta}*r = p^(1/{beta})*m)                  ///
>         (ln(F.m) = {rhom}*ln(m))                     ///
>         (ln(F.z) = {rhoz}*ln(z))                     ///
>         , exostate(z m) observed(p r) unobserved(x)
Solving at initial parameter vector ...
Checking identification ...

First-order DSGE model

Sample: 1955q1 - 2015q4                         Number of obs     =        244
Log likelihood = -753.57131
```

	Coef.	OIM Std. Err.	z	P>|z|	[95% Conf.	Interval]
/structural						
beta	.5146672	.0783493	6.57	0.000	.3611054	.668229
phi	.1659058	.0474002	3.50	0.000	.0730032	.2588083
rhom	.7005483	.0452634	15.48	0.000	.6118335	.789263
rhoz	.9545256	.0186417	51.20	0.000	.9179886	.9910627
sd(e.z)	.650712	.1123897			.4304321	.8709918
sd(e.m)	2.318204	.3047452			1.720914	2.915493

Tests of economic hypotheses

```
. nlcom 1/_b[beta]

       _nl_1:  1/_b[beta]
```

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
_nl_1	1.943	.2957884	6.57	0.000	1.363265	2.522735

Policy questions

What is the effect of an unexpected increase in interest rates?

Estimated DSGE model provides an answer to this question. We can subject the model to a shock, then see how that shock feeds through the rest of the system.

Effect on impact

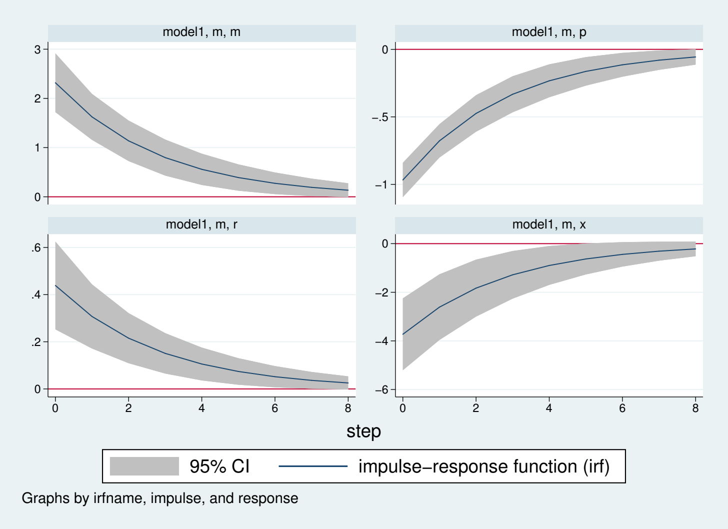
```
. estat policy
Policy matrix
```

	Coef.	Delta-method Std. Err.	z	P>|z|	[95% Conf.	Interval]
x						
z	2.59502	.9077695	2.86	0.004	.8158242	4.374215
m	-1.608216	.4049684	-3.97	0.000	-2.401939	-.8144921
p						
z	.8462697	.2344472	3.61	0.000	.3867617	1.305778
m	-.4172522	.0393623	-10.60	0.000	-.4944008	-.3401035
r						
z	1.644305	.2357604	6.97	0.000	1.182223	2.106387
m	.1892777	.0591622	3.20	0.001	.0733219	.3052335

Effect over time: impulse response functions

```
. irf set nkirf.irf, replace
. irf create model1
. irf graph irf, impulse(m) response(p x r m) byopts(yrescale) yline(0)
```

Impulse responses from the estimated model



Analyzing nonlinear DSGE models

- We can do more than look at impulse responses
- We will switch to a textbook model and explore its features

The stochastic growth model

$$
\begin{aligned}
1 &= \beta E_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{-1} (1 + r_{t+1} - \delta) \right] \\
y_t &= z_t k_t^{\alpha} \\
r_t &= \alpha z_t k_t^{\alpha - 1} \\
k_{t+1} &= y_t - c_t + (1 - \delta) k_t \\
\ln z_{t+1} &= \rho \ln z_t + e_{t+1}
\end{aligned}
$$

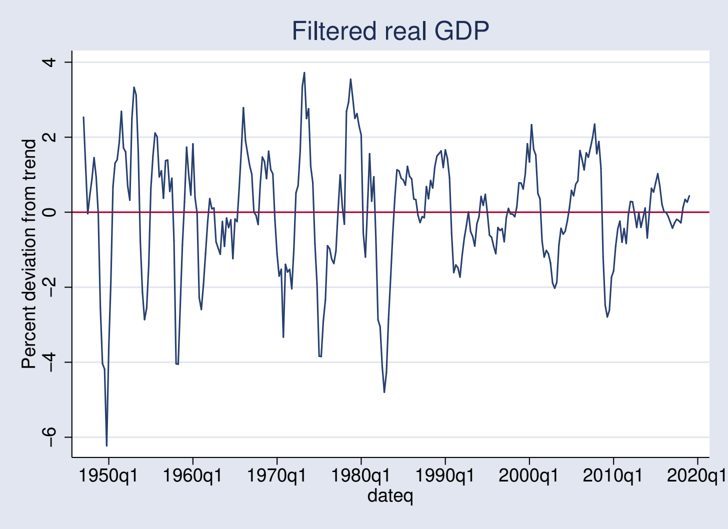
The stochastic growth model in Stata

```
. dsgenl (1={beta}*(c/F.c)*(1+F.r-{delta}))                                   ///
>        (r = {alpha}*y/k)                                                     ///
>        (y=z*k^{alpha})                                                       ///
>        (f.k = y - c + (1-{delta})*k)                                         ///
>        (ln(F.z)={rhoz}*ln(z)),                                               ///
>        exostate(z) endostate(k) observed(y) unobserved(c r)
```

Data

```
. import fred GDPC1
. generate dateq = qofd(daten)
. tsset dateq, quarterly
. generate lgdp = 100*ln(GDPC1)
. tsfilter hp y = lgdp
```

Data



Filtered real GDP
Percent deviation from trend
4
2
0
−2
−4
−6
1950q1
1960q1
1970q1
1980q1
1990q1
2000q1
2010q1
2020q1
dateq

Parameter estimation

```
. constraint 1 _b[beta]=0.96

. constraint 2 _b[alpha]=0.36

. constraint 3 _b[delta]=0.025

. dsgenl (1={beta}*(c/F.c)*(1+F.r-{delta}))                              ///
>        (r = {alpha}*y/k)                                               ///
>        (y=z*k^{alpha})                                                 ///
>        (f.k = y - c + (1-{delta})*k)                                   ///
>        (ln(F.z)={rhoz}*ln(z)), constraint(1/3) nocnsreport             ///
>        exostate(z) endostate(k) observed(y) unobserved(c r) nolog
Solving at initial parameter vector ...
Checking identification ...

First-order DSGE model

Sample: 1947q1 - 2019q1                         Number of obs     =        289
Log likelihood = -362.93403
```

y	Coef.	OIM Std. Err.	z	P>|z|	[95% Conf.	Interval]
/structural						
beta	.96	(constrained)				
delta	.025	(constrained)				
alpha	.36	(constrained)				
rhoz	.8391786	.0325307	25.80	0.000	.7754197	.9029375
sd(e.z)	.8470234	.0352336			.7779668	.91608

After parameter estimation

- Long run behavior: steady–state
- Impact effect of shocks: the policy matrix
- How shocks persist over time: the transition matrix
- Exploring the structure: model-implied covariances
- Dynamic effects: impulse responses

Steady–state

```
. estat steady
Location of model steady-state
```

	Coef.	Delta-method Std. Err.	z	P>|z|	[95% Conf.	Interval]
k	13.94329	.	.	.	.	.
z	1	.	.	.	.	.
c	2.233508	.	.	.	.	.
r	.0666667	.	.	.	.	.
y	2.582091	.	.	.	.	.

Note: Standard errors reported as missing for constrained steady-state values.

Policy matrix

```
. estat policy
```

Policy matrix

	Coef.	Delta-method Std. Err.	z	P>|z|	[95% Conf.	Interval]
c						
k	.6371815	.	.	.	.	.
z	.266745	.0244774	10.90	0.000	.2187701	.3147198
r						
k	-.64	.	.	.	.	.
z	1	.	.	.	.	.
y						
k	.36	.	.	.	.	.
z	1	.	.	.	.	.

Note: Standard errors reported as missing for constrained policy matrix values.

State transition matrix

```
. estat transition

Transition matrix of state variables
```

	Coef.	Delta-method Std. Err.	z	P>|z|	[95% Conf.	Interval]
F.k						
k	.9395996	.	.	.	.	.
z	.1424566	.0039209	36.33	0.000	.1347717	.1501414
F.z						
k	0	(omitted)				
z	.8391786	.0325307	25.80	0.000	.7754197	.9029375

Note: Standard errors reported as missing for constrained transition matrix values.

Model–implied covariances

```
. estat covariance y
```

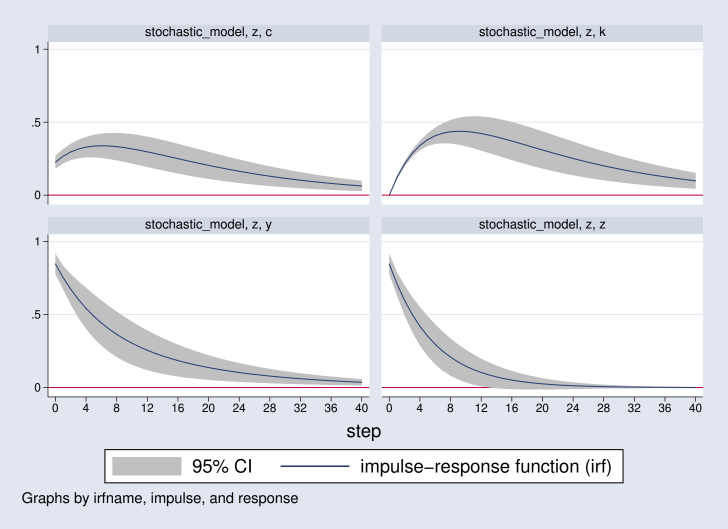
Estimated covariances of model variables

	Coef.	Delta-method Std. Err.	z	P>|z|	[95% Conf.	Interval]
y						
var(y)	3.872087	.9694708	3.99	0.000	1.971959	5.772215

Impulse responses

```
. irf set stochirf.irf, replace
. irf create stochastic_model, step(40)
. irf graph irf, impulse(z) response(y c k z) yline(0) xlabel(0(4)40)
```

Impulse responses



Conclusion

- `dsgenl` estimates the parameters of nonlinear DSGE models
- View steady–state, policy matrix, transition matrix
- View model–implied covariances
- Create and analyze impulse responses

Thank You!