SYMBOLIC TOOLS

Symbolic Tools for GAUSS ECONOTRON SOFTWARE, INC. Version 1.0

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Contents

Chapter 1

Introduction

1.1 Concept

The concept of symbolic manipulation is to augment the numeric and graphical capabilities of GAUSS with additional types of mathematical computations. These include:

- Symbolic Algebra. This includes analytic differentiation and integration, as well as simplification.
- Linear Algebra. This capability allows for the exact (as opposed to the numeric) evaluation of matrix forms, including inverses, determinants and eigenvalues.
- Language Extension. This permits the evaluation of a wide range of mathematical functions and matrix forms, effectively extending the GAUSS language.
- Precision. Numerical evaluation of functions can occur at any specified level of accuracy.

The computational work is carried out by the Maple kernel using the Open Maple interface. Maple is a symbolic mathematics package developed at the University of Waterloo, and distributed by Waterloo Maple, Inc.

Symbolic Tools consists of a set of functions that provide an interface between GAUSS and the Maple kernel. These functions provide the means of sending variables or code from GAUSS to the kernel, running GAUSS code within the kernel, and returning variables back to GAUSS.

GAUSS is a programming language in which each variable is given a name - gnp, coeff, foo – any legal GAUSS name. Associated with each name is a type - a scalar, matrix, string, stringarray, etc. Each variable has associated value(s) - 6.4 for a scalar, "Hello World" for a string, {2,4,6} for a vector, etc. Symbolic Tools adds a new datatype - a symbol - to the set of datatypes used by GAUSS. So in normal GAUSS mode:

$$
z = x + y;
$$

results in z being generated as a scalar taking the value 5 assuming that x is a scalar with the value 2 and y is a scalar with the value 3. The same command in which both x and y are symbols results in z being created as a symbol, with the value $x + y$. Thus in symbolic mode, the same GAUSS syntax is used, but instead of manipulating value or strings, one manipulates symbols. Symbols can represent scalars, vectors or matrices, but the content of these variables does not need to be specified.

As a simple example, if vectors v1 and v2 are specified as:

$$
v1 = {a, b};
$$

 $v2 = {c, d};$

then:

 $v1'v2 = a*c + b*d$

The ability to use symbolic arithmetic enables a GAUSS optimization process to use analytic gradients and Hessians — this is called automatic differentiation. There exists a number of GAUSS optimization packages — Maxlik, Optmum, CML, etc — that require the gradient and Hessian of the function being optimized in order to evaluate the appropriate search direction. In the default, these are calculated numerically using forward differencing:

$$
df/db_i = [f(b_1,..b_i + h,..b_n) - f(b)]/h
$$

The Hessian is similarly calculated using the second derivative. For the Hessian, the number of function evaluations increases quadratically with the number of parameters. Thus, as the gradients and Hessians have to be recalculated at each iteration, this process becomes very time intensive if there are a large number of parameters.

These optimization packages provide mechanisms whereby the user can specify the procedures that will return values of the gradient and/or Hessian, instead of doing forward differencing. Thus, as a trivial example, if the function were:

$$
F = ln(b);
$$

then the user could specify functions for the gradient and Hessian:

$$
GF = 1/b;
$$

$$
HF = -1/b^2;
$$

Symbolic Tools can create GAUSS procs for the gradients and Hessian of the specified function, and this approach works very well for enabling automatic differentiation in GAUSS. Using Monte Carlo simulation of a Tobit example with 2000 observations and 11 parameters, the AD gradient took 10% of the time required for forward differences using gradp - ie. approximately a 10 fold speed improvement. Similar results were also obtained for the Hessian, with the additional advantage that the AD methodology generated much more precise estimates of the gradients and Hessian.

1.2 Symbolic Modes

Symbolic operations are carried out using the Maple kernel. There are two main modes utilized:

1. Direct Mode: Symbolic code is sent to Maple, where the symbolic operations are stored. Values are then sent to Maple, the symbolic operations are carried out using these values, and the numeric data is returned to GAUSS.

2. Compiled Mode: Symbolic code is sent to Maple, which then creates a GAUSS proc which replicate the symbolic operations carried out in Maple.

A trivial example demonstrates both ideas. We wish to evaluate the gradient of the function $sin(x * y)$ at the point $x = .5, y = .75$.

txt = " $slist = {x,y};$ $llf = sin(x*y);$ llfg = gradp(llf,slist); $"$;

Using this direct mode method, this chunk of code is executed using the symrun command, the values for x and y are sent to the Maple kernel, and the results retrieved.

```
call symrun(txt);
call symput(x, "x");
call symput(y,"y");
rslt = symget("llfg");
```
The symrun command generates (in Maple) the symbolic variables slist, llf and llfg. llfg is a symbolic vector:

llfg = { $y * cos(x * y)$, $x * cos(x * y)$ }

We evaluate the value llfg takes at $x = .5, y = .75$ by specifying the values of the symbols x and y using the symput statement, and retrieving the new value of llfg using the symget statement.

In compiled mode, the same code is executed using the symproc command:

```
call symproc("diffsin","x,y","llfg",txt);
```
This generates a GAUSS procedure called diffsin as a string, with an input argument x,y , and creating an output ι life. The procedure is also compiled, and can be immediately called as a standard GAUSS proc.

```
proc diffsin(x,y);
local t0, t2, unknown;
unknown = zeros(2,1);
t2 = cos(x + y);unknown[1+0] = sumc ( t2 . * y) ;unknown[1+1] = sumc( t2 .* x) ;
retp(unknown);
endp;
```
Thus the difference between the two modes is that in Direct Mode, the numeric evaluation takes place under Maple, while under Compiled Mode it takes place under GAUSS. The latter is between 10 and 100 times faster, and thus for most operations where speed is essential, such as automatic differentiation, Compiled Mode is more suitable. The two example files intro1.e and introl2.e in the symbolic\samples\tutorial folder demonstrate these two modes.

Besides symbolic analysis, Symbolic Tools can be used to augment GAUSS functionality by using Maple commands that are not available in GAUSS. To take a trivial example, one wishes to know the value of the norm of matrix. This is undertaken using the symmaple statement:

```
x = \text{rndu}(3, 4);\texttt{xnorm} = \texttt{symmaple("norm(x)", 0)};
```
These examples are coded in the file symbolic\samples\tutorial\into3.e.

1.3 Example files

The symbolic\samples folder contains a large number of example files that demonstrate the capabilities of Symbolic Tools.

- Tutorial These files are described above the new user should start with these files, before progressing to the other examples.
- GAUSS Each file in this folder demonstrates how the specified GAUSS command is used within Symbolic Tools.
- AD The Automatic Differentiation folder contains two subfolders (Maxlik and Optmum). See the file readme.txt which describes AD, and the difference between the two folders.
- Applications A number of applications, including integration and non-linear estimation.

Chapter 2

Installation and Testing

This chapter describes the hardware and software configuration required to run Symbolic Tools on your computer.

2.1 Installation Requirements

The Symbolic Tools (vsn. 5) system requirements are:

- Windows 9x, NT4, ME, 2000, or XP.
- GAUSS for Windows 4.0 or higher, or the GAUSS Engine for Windows 4.0 or higher.
- Maple 9 for Windows or higher. An evaluation version of Maple 9 available at http://register.maplesoft.com/TrialDownload.asp

2.2 Installing Symbolic Tools

Before you open the product package, please read the license agreement that accompanies Symbolic Tools. By installing and using the product, you accept the terms of this agreement.

The program files on the CD are compressed, so you cannot simply copy them to your computer. Rather, you must run the installation program which decompresses the files and copies them to your hard disk in the appropriate directories.

INSTALLATION AND TESTING

- 1. Insert the Symbolic Tools CD into the appropriate drive.
- 2. From the Windows Start menu, chose Run.
- 3. Type d:\setup.exe (where d: is the letter for your CD drive).
- 4. Choose OK.
- 5. Follow the instructions on the screen.

The installation routine creates a folder called symbolic on the GAUSS or GAUSS Engine folder. Within this folder, the following subfolders are created

2.3 Testing Symbolic Tools

Launch GAUSS (or engauss.exe for the GAUSS Engine) and run the file: symbolic\samples\tutorial\symtest.e.

This opens the Maple kernel, and reports some statistics about the kernel. There are three other files in the tutorial folder - intro1.e, intro2.e and intro3.e, which you should also run to check that the installation has been correctly performed. These files also demonstrate programming techniques in Symbolic Tools.

Chapter 3

Tutorial

GAUSS is a numeric based application, in that it operates on variables that have specified values, such as $x = 2, y = 4$ or $z = "abc"$. Thus $x * y$ results in 8. Symbolic applications - such as Maple - do not operate solely on numerical values - they can in addition, act on symbolic values. So if $x = a$, and $y = b$, then $x * y$ results in $a * b$. This can be very useful - for example, we might want to have the indefinite integral of x^2 , but not yet want it evaluated. So we would like to have this indefinite integral available as $x^3/3$.

Symbolic Tools makes these symbolic results available to GAUSS as procedures. Four examples are presented below:

3.1 Example 1

We require the determinant of a matrix where the elements on the principal diagonal are symbolic. Obviously this could be evaluated using GAUSS, but the idea might be that one had a huge matrix, and one wanted to know the determinant if only a couple of elements change.

The GAUSS program to do this is fairly simple, and is shown below in the string txt. Besides the GAUSS command symmat, which is used to define a symbolic matrix, the code in txt is conventional GAUSS . The symproc command is used to create a GAUSS procedure — this command takes the procedure name, the

procedure input arguments, the procedure output arguments and the procedure code as arguments. In this example, symproc takes the code in the string txt, creates a GAUSS proc called symdet, and compiles it. It is then called with numeric arguments in the standard manner. Thus Symbolic Tools uses procedures to map symbolic arithmetic to numerical output.

```
*******************************************************************
The GAUSS program:
library symbolic;
call symstate(reset);
proc symdet; endp;
txt = "x11 = x[1];x22 = x[2];\text{amat} = \text{symmat}(2,2,\{x11,2,8,x22\});
      rslt = det(amat);
      ";
call symproc("symdet","x","rslt",txt); // compile the procedure
let xdiag = 19; // some intial values
rslt = symdet(xdiag); // call the compiled proc
"rslt \n" rslt;
*******************************************************************
The Output:
rslt
     -7.0000000
*******************************************************************
The computer generated proc:
proc symdet(x);
local t0, t2, unknown;
     t2 = x[1+0] .* x[1+1]-16.0;retp( t2 );
endp;
*******************************************************************
```
3.2 Example 2

We require the analytic gradient of a function. The example below shows a simple example, but this provides the basis for automatic differentiation.

The GAUSS program to do is shown below. The string txt creates the gradient of the function $sin(x*y)^2$, which is then created as a proc called diffsin. Note both that gradp is overloaded, so that it can accept a symbol list (slist) instead of numeric values. Also note that the proc allows arguments that are both scalars and matrices.

```
*******************************************************************
The Gauss program:
library symbolic;
proc diffsin; endp; \frac{1}{2} // dummy proc
call symstate(reset); // symbolic reset
txt = "slist = \{x,y\};
      llf = sin(x*y)^2;llfg = gradp(llf, slist);";
call symproc("diffsin","x,y","llfg",txt); // compile the proc
rslt = diffsin(0.5, 0.75); // call the proc
 "\n rslt " rslt;
*******************************************************************
The Output:
rslt
     0.51122907
     0.34081938
*******************************************************************
```

```
The computer generated proc:
```

```
proc diffsin(x,y);
local t0, t1, t2, t3, t4, unknown;
     unknown = zeros(2,1);
     t1 = x . * y;t2 = \sin(t1):
     t3 = cos(t1);
     t4 = t2 .* t3;
     unknown[1+0] = sumc( 2.0 .* t4 .* y) ;
     unknown[1+1] = sumc( 2.0 .* t4 .* x) ;
     retp(unknown);
endp;
```
3.3 Example 3

We require the indefinite integral of a function. The example below shows a very simple example, but much more complicated cases exist.¹ The GAUSS program to do this is shown below. The txt provides the integral of $y * x^2$ (which is not very exciting) as a proc called intsim. Intquad is used - we don't need intquad1, intquad2, etc. since the level of integration is given by the length of slist. Again, intquad is overloaded to permit both numeric and symbolic integration.

```
*******************************************************************
The Gauss program:
library symbolic ;
proc intsim; endp; \frac{1}{2} // dummy proc
call symstate(on); \frac{1}{2} // symbolic reset
 txt. = "
       slist = {x,y};l1f = y*(x^2);intg = intquad(llf,slist);
 \overline{1}
```
This technique was used to create the integral of the Pearson function in "Simulated Latent Variable Estimation of Models with Ordered Categorical Data", (J. Breslaw and J. McIntosh) Journal of Econometrics, 87, 1998, pp 25-47.

```
";
call symproc("intsim","x,y","intg",txt); // compile the proc
rslt = intsim(0.75, 0.5); // call the proc
"rslt \n" rslt;
*******************************************************************
The Output:
rslt
    0.017578125
*******************************************************************
The computer generated proc:
proc intsim(x,y);
local t0, t1, t2, t5, unknown;
    t1 = y . * y;t2 = x . * x;t5 = t1 .* t2 .* x ./ 6.0;
    retp( t5 );
endp;
*******************************************************************
```
3.4 Example 4

Finally, a real world example - a Tobit estimation. The code shown is a Monte Carlo simulation of the Hessian, based on 2000 observations, 4 parameters, and 200 replications. The results show element [2,1] of the Hessian for the first 10 replications, while the time in each case is for the full 100 replications. As can be seen, the symbolic code is over 10 times faster than hessp.

```
*******************************************************************
/* Symbolic example: Tobit.e
      proc tobit(y,indx,sigma);
      y is the censored variable
       indx is the vector of the index
       sigma is the parameter for the std. error of the residual
```
*/ /** ** Initialization ** **/ library symbolic;
call symstate(reset); // Initialize Symbolic tools
... call symdebug(off); // Debug mode shows line by line rndseed 12345; mode = 1; $\frac{1}{2}$ // 0 - gradient; 1 - Hessian replics = 200; $\frac{1}{2}$ // number of replications num = 2000; $\frac{1}{\sqrt{mn}}$ number of observations // create the data $signa = 2;$ x mat = ones(num,1) x ndu(num,2); $y = 2*signa*rndn(num,1);$ $x1 = xmat[.,1];$ $x2 = x \text{mat}$ [.,2]; x3= xmat[.,3]; $xrnd = rndu(replies, 4);$ // parameters rslt = zeros(replics, 2); $\frac{1}{1}$ initialization clear time1, time2; $ji = 2;$ title = "Tobit process"; process = "Gradient" \$| "Hessian"; cls; print /flush title ; /** ** Code for a Tobit process ** **/ // GAUSS code as proc proc llfn(bpar); local llf1, indx, bvec, sigma, llf2, llf, h;

```
h = .000001;bvec = bpar[1:3];
  signa = bpar[4];\text{index} = \text{rand*}bvec;
  signa = maxc(sigma|h);
  llf1 = -((y-indx)^2) / (2*sigma) - .5*ln(2*pi*sigma);llf2 = ln(h + cdfnc(intx/sqrt(sigma)));
  llf = (y .gt 0).*llf1 + (y .le 0).*llf2;retp(sumc(llf));
endp;
// Gauss symbolic representation as string
txt = "h = .000001;
    slist = symset(bpar,b,4);indx = x1*b1+x2*b2+x3*b3;signa = maxc(symvec(\{h,b4\}));llf1 = -(y-indx)^2 / (2*sigma) - .5*ln(2*pi*sigma);llf2 = ln(h+cdfnc(indx/sqrt(sigma)));
    llf = sumc((y .gt 0).*llf1 + (y .le 0).*llf2);llfg = gradp(llf, slist);llfh = hessp(llf,slist);";
/********************************************************************
** create a GAUSS proc that does the AD **
********************************************************************/
proc llfproc; endp; \frac{1}{2} // dummy proc
if mode == 0;call symproc("llfproc","bpar","llfg",txt); // gradient
else;
  call symproc("llfproc","bpar","llfh",txt); // hessian
endif;
/********************************************************************
** Monte Carlo using forward difference **
********************************************************************/
```
cls;

```
print /flush title ;
print /flush
  "\n Evaluating " process[mode+1] " using Forward Differencing...";;
d1=date;
j = 1;do while j \leq replics;
    if mode == 0:
       gvec = gradp(k11fn, xrnd[j,.]');else;
       gvec = hessp(kllfn, xrnd[j, .]');endif;
    rslt[j,1] = gvec[1,jj];j = j+1;endo;
time1 = ethsec(d1, date)/100;/********************************************************************
** Monte Carlo using symbolic code **
********************************************************************/
print "";
print /flush
  "\n Evaluating " process[mode+1] " analytically...";;
d1=date;
j = 1;do while j <= replics;
    x = vec(xrnd[j, .]),gvec = 11fproc(x);rslt[j,2] = gvec[1,jj];j = j+1;endo;
time2 = ethsec(d1, date)/100;/********************************************************************
** Monte Carlo results ** **
********************************************************************/
cls;
print /flush title ": " process[mode+1];
\frac{0}{0} \frac{0}{1};
        numerical analytical ";
rslt[1:10,.];
```
$\it TUTORIAL$

```
" \nTime (secs)\n " time1~time2;
" \nSpeed factor: " time1/time2;
" \in \mathbb{R}^n;
```

```
*******************************************************************
```
The Output:

 $\it TUTORIAL$

Chapter 4

Symbolic Data Types and **Operations**

As in GAUSS, each variable refers to an entity - these can be individual scalars, or more complex groupings such as vectors and matrices. In GAUSS, each element is given a value, such as $x = 4.7$; for a numeric component, or $txt = "abc";$ for a string component. In symbolic mode, the same GAUSS syntax is used, but instead of manipulating value or strings, one manipulates symbols. Symbols can occurs as elements, or as compontents of a list, a vector, or a matrix.

4.1 Data types

Symbolic Tools supports the following data types:

scalar (real, complex, symbol) vector (real, complex, symbol) matrix (real, complex, symbol) string

The following types are not supported:

arrays string arrays structures GAUSS data sets band and sparse matrices character vectors date and time types graphics fuzzy operators

4.2 Symbolic Elements

Each symbol is designated by a name, which is just the GAUSS variable name x, y, foo - any legal GAUSS name. Thus the command:

sigma = b;

defines a GAUSS variable, sigma which has a value of b. b is neither numeric, nor a string; rather it is just a symbol.

4.3 Symbolic List

The main structure used in the symbolic arithmetic is a list - an ordered list of symbols:

 $slist = {a,b,c,d};$

These lists are the basis for most symbolic operations.

List Creation Lists can be created in a number of ways:

- 1. slist = ${a,b,c,d}$;
- 2. slist = symlist(4,x); This creates slist = ${x1, x2, x3, x4}$;
- 3. slist = symlist(vect); This converts a vector to a list.
- 4. slist = symset(bpar, b, 4); This creates slist = ${b1, b2, b3, b4}$ and assigns each symbol to the corresponding element of the GAUSS vector bpar.

5. slist = symdat(dta,b,4); This creates slist = ${b1, b2, b3, b4}$ and assigns each symbol to the corresponding column of the GAUSS matrix dta.

Lists are used in creating symbolic vectors and matrices (see below), and as arguments of functions that evaluate accross a list, such as symbolic differentiation and integration.

Element Identification

slist = ${a,b,c,d}$; slist[3] is 'c' slist[2:3] is {b,c}

List Concatenation If vlst1 and vlst2 are two lists, then concatenation occurs by:

```
vlst1 | vlst2;
vlst1 ~ vlst2;
symlist({vlst1,vlst2}).
```
List Operations

```
blst = {b1, b2, b3};x1st = {x1, x2, x3};blst+xlst \rightarrow {b1+x1,b2+x2,b3+x3}
blst.*xlst \rightarrow {b1*x1,b2*x2,b3*x3}
blst*xlst \rightarrow b1*x1+b2*x2+b3*x3
```
4.4 Symbolic Vector

A vector consisting of symbols is defined as:

```
v =symvec({x,y,z});
```

```
or
```
 $slist = {x,y,z};$ $v =$ symvec(slist);

All symbolic vectors are symbolic matrices with a single column. Thus all the operations applicable to symbolic matrices apply to symbolic vectors.

4.5 Symbolic Matrix

A 2x3 matrix consisting of symbols is defined as:

$$
m = \text{symmat}(2,3,\{a,b,c,d,e,f\})
$$

or

 $mattst = {a,b,c,d,e,f};$ $m =$ symmat $(2,3, \text{mattist})$;

Symbolic matrix operations closely resemble numeric operations:

Element Identification

```
xmat = matrix(2,2,\{a,b,c,d\});xmat[2,1] is 'c'
```
Concatenation

 $q = \{a,b,a\} | \{b,b,c\};$ $v1 =$ symmat $(1, 2, \{a, b\})$; $v2 =$ symmat $(1, 2, \{b, b\})$; $v3 =$ symmat $(1, 2, \{a, c\})$; $v = v1|v2|v3;$ $m = \{a\}^{\sim} v1$;

Transpose $m = x'$; You may need to use (x') to get the precedence correct.

Transpose Multiply $xx = x' * x$ (not $x'x$).

- Element Operations .∗ and ./ work as expected but may need parenthesis to get precedence correct.
- Matrix Multiplication $x * y$ works as in GAUSS if the type of x and y are known. Thus if x and y are defined by a symmat statement, then there is no problem. If x and y are symbols, then it is not clear whether these symbols represent scalars or matrices. In the default, when x and y are both symbols, $x * y$ generates $x * y$. To force matrix multiplication, use the Maple syntax $x \& y$. $x' * y$ is always taken as matrix multiplication. Example:

bhat = $inv(x'*x)$ &* $(x'*y)$;

x and y are symbols, so they could be matrices, or scalars. The $x' * x$ term is recognized as requiring a matrix type mulitplication, since this operation occurs almost exclusively on matrices. However, $inv(x' * x)$ multiplied by $(x' * y)$ is a symbol times a symbol, so to ensure the required matrix multiplication, we use the &∗ format.

Matrix Division x/y works as in GAUSS if the type of x and y are known. If x and y are symbols, then it is not clear whether these symbols represent scalars or matrices. In the default, when x and y are both symbols, then x/y generates $x \, y$. To force the matrix interpretation of x/y , use the syntax $x\&y.$

Other Operations The following work as in GAUSS :

Logic Operations GAUSS assumes that true $= 1$ and false $= 0$. Maple does not use this rule. Symbolic Tools evaluates dot functions (., ., .and etc) so as to return unity or zero as expected. Relational and boolean operators (non dot) are used for flow control, and return true or false.

Chapter 5

Symbolic Tools Commands - Summary

5.1 Summary

5.1.1 GAUSS Commands

These commands are called from GAUSS.

SYMBOLIC TOOLS COMMANDS - SUMMARY

5.1.2 Maple Commands

These commands are called from within a string sent to the Maple kernel.

Chapter 6

Reference - GAUSS commands

These commands extend the GAUSS language to allow for control over the Maple kernel and symbolic manipulation. Symbols require a new data type - a list and these commands provide support for this data type.

SYMARG REFERENCE - GAUSS COMMANDS

Purpose

Specifies data argument in symproc.

Format

SYMARG(argn);

Inputs

argn literal, argument number.

Remarks \blacksquare

> When using the symproc command, the user specifies the input arguments that are required by the proc. Normally, no additional knowledge is required. However, when one of the arguments is data, symproc needs to know this to undertake the correct dimensions of the output of the proc. The argument number of a parameter that is data is supplied in argn.

The default value (2) is set in symbolic.dec.

Example

```
library symbolic;
  ...
call symarg(3);
call symproc("myproc","avec,bvec,dta","llf",txt);
```
In this example, the third input argument of myproc is data.

■ See also

SYMPROC

REFERENCE - GAUSS COMMANDS SYMDEBUG

Purpose

Controls debugging mode of Symbolic Tools.

■ Format

SYMDEBUG(mode);

Inputs

 $mode$ literal, debug mode $(on, [off]).$

Remarks \blacksquare

> In normal operation, code is sent to the Maple kernel, and results are retrieved from the kernel with no output from the kernel unless an error is detected. Turning the debug mode to on results in the kernel displaying each line of Maple code as it is executed, and displaying any file created with symproc.

The default mode is off; this is set in symbolic.dec.

Example \blacksquare

library symbolic; call symdebug(on);

In this example, the debug mode is enabled.

SYMGAUSS REFERENCE - GAUSS COMMANDS

Purpose

Sends a GAUSS command to the kernel for execution.

Format

 r slt = SYMGAUSS (txt) ;

Inputs

txt string, GAUSS command

Outputs

rslt GAUSS output

Remarks

The symgauss command executes the single GAUSS command embedded in the string txt in the Maple kernel, and returns the result. The arguments to the GAUSS command are automatically satisfied from the GAUSS workspace.

This facility permits the user to execute a GAUSS command under Maple, with input and output managed seamlessly. Note that Maple is case sensitive, so the code in txt should be lower case. Only GAUSS commands with single returns are permitted.

Example

```
library symbolic;
...
let x[3,3] = 4 2 6 8 5 7 3 8 9;rslt = symgauss("cond(x)");
"The result is: " rslt;
```
In this example, the GAUSS matrix x is used in the symgauss command to derive the condition number of x using the Maple kernel. The result is returned and displayed by GAUSS.

REFERENCE - GAUSS COMMANDS SYMGET

Purpose

Retrieves a variable from the Maple kernel.

Format

 r slt = SYMGET (name) ;

Inputs

name string, variable name

Outputs \blacksquare

rslt GAUSS variable

Remarks \blacksquare

> The symget command retrieves a variable from the Maple kernel, and stores it in the GAUSS workspace. Valid data types are Maple matrices, vectors, scalars and strings. Numeric data is stored as regular GAUSS variables, while a symbolic result is stored as a string. Memory allocation and type recognition are taken care of automatically.

Example

```
library symbolic;
 ...
rslt = symget("llfg");
"The result is: " rslt;
```
In this example, a Maple variable, llfg, is retrieved and displayed by GAUSS.

SYMHELP REFERENCE - GAUSS COMMANDS

Purpose

Displays the Symbolic Tools online help

Format

SYMHELP ; SYMHELP (topic);

Inputs

topic literal, Maple topic

Remarks \blacksquare

The symhelp command with no arguments displays the online help file. If $topic$ is specified, the appropriate Maple help file is displayed in Notepad.

Example

```
library symbolic;
symhelp;
symhelp (linalg);
symhelp codegen;
```
The first call to symhelp brings up the online help facility. The second and third calls show alternative methods of retrieving the Maple help page for linalg and codegen respectively.

REFERENCE - GAUSS COMMANDS SYMMAPLE

■ Purpose

Sends a Maple command to the kernel for execution.

Format

```
rslt = SYMMAPLE (txt, mode) ;
```
Inputs

Outputs

rslt GAUSS output

Remarks

The symmaple command executes the Maple command embedded in the string txt in the Maple kernel, and returns the result. The arguments to the Maple command are automatically satisfied from the GAUSS workspace.

mode determines how the arguments are treated. mode is set to zero for those cases where the entire argument is treated as a whole, such as the determinant of a matrix. mode is set to unity for those cases where the function is evaluated on each element of the argument, such as sin.

This facility permits the user to execute a Maple command under Maple, with input and output managed seamlessly. Note that Maple is case sensitive, so the code in txt should be correctly cased for Maple. Only Maple commands with single returns are permitted.

Example

library symbolic; ... let $x[3,3] = 4 2 6 8 5 7 3 8 9;$ r slt = symmaple(" r atform (x) ", 0); "The result is: " rslt;

In this example, the GAUSS matrix x is used in the symmaple command to derive the rational canonical form (or Frobenius form) of x using Maple. The result is returned and displayed by GAUSS.

SYMMODE REFERENCE - GAUSS COMMANDS

Purpose

Controls the Symbolic Tools syntax.

Format

SYMMODE (mode) ;

Inputs

mode literal, operating mode ([Gauss], Maple)

Remarks \blacksquare

> In the default, GAUSS code is parsed to Maple format prior to being sent to the Maple kernel. Pure Maple code can be also sent to the kernel, in which case no parsing is required. The default mode (GAUSS) is set in symbolic.dec

Example

```
library symbolic;
call symmode(maple);
txt = " with (linalg);
        x:=matrix(2,2,[1,2,3,4]);
        z:=trace(x); ";call symrun(txt);
```
In this example, parsing is turned off, so the raw code is sent to Maple as is.

$\begin{minipage}{0.4\linewidth} \emph{REFERENCE} - \textit{GAUSS}\ \textit{COMMANDS} \end{minipage}$

Purpose

Controls the Symbolic Tools output buffer.

Format

SYMOUT (mode) ;

Inputs

mode literal, output mode

Remarks \blacksquare

This command provides a diary of the Maple output. mode takes the following values:

Example

```
library symbolic;
call symout(reset);
   ...
call symout(view);
```
In this example. the output buffer is cleared, some commands are carried out, and the buffer is then viewed using Notepad.

SYMPROC REFERENCE - GAUSS COMMANDS

\blacksquare Purpose

Creates and compiles a GAUSS proc derived from symbolic code.

Format

 $\text{pre} = \text{SYMPROC}($ *pname, inarg, outarg, txt*);

Inputs

Outputs

prc string, GAUSS proc

Remarks

The symproc command creates and compiles a GAUSS proc based on symbolic code evaluated by the Maple kernel. In a trivial example, if one had a function $sin(x)$, and wished to have access to a proc that was the gradient of this function, then that proc would simply return $cos(x)$. symproc automates this process. The GAUSS code is parsed, sent to the Maple kernel, and executed. Maple takes the required value (say a gradient), and returns the optimized code to create this gradient. Symproc reparses this optimized code to be a GAUSS compatible proc, and compiles the proc. This proc is then accessible for use by the user.

The idea behind writing code for a symbolic process is to compose the code based on a single (symbolic) observation. Symbolic Tools takes care of creating a proc for n observations. Since all symbols will become matrices when the proc is run under GAUSS, all operations will be dot evaluated.

In most cases, a single argument (a parameter vector) is passed to the compiled proc. Symbolic Tools is usually able to ascertain all the information needed from the code itself. An exception is where data is passed to the proc as an argument, in which case one should use the symarg command to specify which argument is data.

For the most part, the Maple kernel will evaluate GAUSS commands as Maple code. Note that Maple is case sensitive, so the code in txt should be lower case. However, you can force a GAUSS evaluation of a GAUSS command by capitalizing the command.

REFERENCE - GAUSS COMMANDS SYMPROC

```
■ Example
```

```
library optmum, symbolic;
optset;
call symstate(reset);
  \ldots load data y, x1 - x5
proc fct(avec);
 local indx, llf;
 indx = avec[1] + avec[2]*x1 + avec[3]*x2 + avec[4]*x4^avec[5]
      + avec[6]*ln(x5+avec[7]);
 llf = sumc((y-indx)^2);retp (llf);
endp;
txt = "slist = symset(bpar,a,7);indx = a1 + a2*x1 + a3*x2 + a4*x4^a5 + a6*ln(x5+a7);
  llf = sumc((y-indx)^2);llfg = gradp(llf, slist);llfh = hessp(llf,slist);
";
proc gradproc; endp;
proc hessproc; endp;
gcode = symproc("gradproc","bpar","llfg",txt);
hcode = symproc("hessproc","bpar","llfh",txt);
\text{log}dprc = &gradproc;
__ophsprc = &hessproc;
\text{avec0} = \text{ones}(7,1);{x, f, g, retcode} = optimum(<i>ktct</i>,<i>avec</i>0);
```
This example shows how one would estimate a non-linear least squares problem using the GAUSS optmum command. The libraries are specified, and each package is set. Optmum requires pointers to procedures that return the function to be minimized, and optionally the gradient and Hessian. The function is specified in fct, which requires 7 parameters, and the data (y, x1-x5) is in core. The equivalent symbolic GAUSS code is supplied as a string (txt), augmented with the code to generate the gradient and Hessian. GAUSS procs for the gradient and Hessian are generated by symproc, as shown. Note the dummy procs for

$\small \textbf{SYMPROC} \qquad \qquad \textit{REFERENCE - GAUSS COMMANDS}$

the gradient and Hessian — these are needed so that GAUSS can compile the program. Note also that the code for these procs is returned in gcode and hcode respectively, so it is easy to cut and past the code into your program if the model specification does not change - ie. once Symbolic Tools has generated the gradient and Hessian, it doesn't need to be run again. Optmum requires that pointers to user supplied gradients and Hessians are placed in __opgdprc and $\overline{}$ _ophsprc respectively. A starting value for the parameters is specified, and the optimization is carried out by Optmum.

$\begin{minipage}{0.9\linewidth} \emph{REFERENCE - GAUSS COMMANDS} \end{minipage}$

Purpose

Sends a variable to the Maple kernel.

Format

SYMPUT (var, name) ;

Inputs

Remarks

The symput command send a GAUSS variable var to the Maple kernel, and stores it under the name vname. The supported data types are shown below.

Supported

Example

```
library symbolic;
call symstate(reset);
x = \text{rndu}(5, 2);call symput(x,"xmat");
```
In this example, the GAUSS matrix x is sent to the Maple kernel and stored under the name xmat.

SYMRUN REFERENCE - GAUSS COMMANDS

Purpose

Executes GAUSS or Maple code using the Maple kernel,

Format

SYMRUN (txt);

Inputs

txt string, code

Remarks

The symrun command executes the code embedded in the string txt by the Maple kernel. If symmode has been set to GAUSS, the code is first parsed. No output is returned unless an error is trapped, or unless symdebug has been set to ON. Note that Maple is case sensitive, so the code in txt should be lower case.

This facility permits the user to execute GAUSS code under Maple. This can be used to allow for symbolic operations in GAUSS, to allow for grater precision than is available in GAUSS, and to extend the GAUSS language through the use of Maple commands.

Example

```
1. library symbolic;
      ...
      txt = "x = symmat(2,2,\{a,b,c,d\}); detx = det(x);";
      call symrun(txt);
      detx = symget("detx");
      "The result is: " detx;
2. txt1 = "fx= (c1-x)/(c0-c1*x+c2*(x^2));\text{intg} = \text{intquad}(\text{fx}, \{x\});
      ";
      call symout(on);
      call symrun(txt1);
      call symout(view);
```
 $\begin{minipage}{0.9\linewidth} \emph{REFERENCE - GAUSS COMMANDS} \end{minipage}$

```
3. let a[3,3] = 1 2 3 1 2 3 1 5 6;
      call symput(a,"a");
      call symrun("ca = charpoly(a, x);");
      ca = symget("ca");"The charpoly of a is " ca;
```
In the first example, the determinant of the symbolic matrix x is evaluated by the kernel, and retrieved and displayed by GAUSS. In the second example, a Pearson function is defined, and the symbolic integral is evaluated by the kernel, and then displayed using the viewer. In example 3, the GAUSS language is extended by using the Maple charpoly function.

$\small \textbf{SYMSTATE}\qquad \qquad \textit{REFERENCE - GAUSS COMMANDS}$

Purpose

Controls the Symbolic Tools environment.

Format

SYMSTATE (mode);

Inputs

mode literal, mode (on, off, reset)

Remarks \blacksquare

mode can take three values:

This command is required to initiate the Symbolic Tools environment. Once loaded, the Maple kernel records all Symbolic Tools activity. The reset mode clears the Maple workspace (like new in GAUSS). A session can be started with either mode set to on or reset.

Example

library symbolic; call symstate(reset);

In this example, the Maple kernel is initialized at the beginning of a session.

REFERENCE - GAUSS COMMANDS SYMTEST

■ Purpose

Provides a mechanism for testing AD code.

Format

proc_f, proc_g, proc_h = SYMTEST (\mathcal{E} fg, \mathcal{E} fs, param, dta);

Inputs

Outputs \blacksquare

Remarks

To use automatic differentiation from within an optimization program, such as Maxlik, one needs to define procedures that takes a parameter argument, and returns the gradient and Hessian respectively. The Symbolic tools symproc command is used to create these procedures. However, before running the estimation, one wants to make sure that the symbolic procedures are correct. symtest does this.

fs is a GAUSS proc that provides symtest with the information needed to test the symbolic code. *fs* returns 5 elements:

symtest evaluates the symbolic function, gradient and Hessian, and prints out the results along with the comparable GAUSS estimates (based on forward differencing) for validation. Strings containing the symbolic code as GAUSS procs for the function, gradient and Hessian are returned.

Example

See the example file ADTest.e in the symbolic\samples\AD folder.

$\small \textbf{SYMTEST} \normalsize \begin{tabular}{p{2.5cm}p{2.5cm}p{2.5cm}p{2.5cm}p{2.5cm}p{2.5cm}} \hline \textbf{S1} & \textbf{S2} & \textbf{S3} & \textbf{S4} & \textbf{S5} & \textbf{S5} & \textbf{S5} & \textbf{S6} & \textbf{S6} & \textbf{S6} & \textbf{S7} & \textbf{S7} & \textbf{S7} & \textbf{S7} & \textbf{S8} & \textbf{S7} & \textbf{S8} & \textbf{S7} & \textbf{S8} & \textbf$

Chapter 7

Reference - Maple commands

These commands extend the GAUSS language to allow for symbolic manipulation. Symbols require a new data type - a list - and these commands provide support for this data type. The following commands are only applicable within the Maple kernel, and are used as part of a string that is sent to the Maple kernel.

SYMDAT REFERENCE - MAPLE COMMANDS

Purpose

Assign symbolic names to columns of a matrix.

Format

 $slst = SYMDAT(m. col):$ $slst = SYMDAT$ (*m, sn, col*);

Inputs

 m literal, matrix name. sn literal, symbol col numeric, column or range

Outputs

slist list

Remarks

The symdat command assigns symbols to specified columns of a data matrix. This facility permits the data to be entered as an argument to the procedure created by the symproc command. Typically, if a vector x is required within a procedure, then specifying the vector as the symbol "x" will permit GAUSS to pick up the global vector x when the procedure is executed. However, MAXLIK requires that both the parameter vector and the data be arguments to procedures that return gradients and Hessians, and symdat provides the required functionality.

Example

```
1. y = symdat(dta, 1);
2. xlist = symdat(data, x, 2:4);indx = x1*b1+x2*b2+x3*b3;
3. xlist = symdat(xmat, v, 5);
```
The first example assigns the symbol y to the first column of the matrix dta. The second example creates three symbols $- x1$, $x2$, $x3 -$ corresponding to columns 2, 3 and 4 of dta respectively. xlist is a list of these three symbols - ie xlist $=$ ${x1,x2,x3}$. The creation of an index is also shown in this example. In the third example, five symbols are created (v1 through v5) corresponding to the the first five columns of xmat.

See also

SYMPROC

REFERENCE - MAPLE COMMANDS SYMEVAL

Purpose

Evaluates a Maple function that expects algebraic inputs.

Format

 $rlst = SYMEVAL$ (fn, arg1,arg2..);

Inputs

Outputs

slist result

Remarks \blacksquare

The symeval command is a utility function that allows the evaluation of a Maple command that normally requires algebraic input. This can always be achieved using the Maple map, map2 and zip commands, but symeval makes it easy.

■ Example

 $x =$ symmat $(2, 2, \{a, b, c, d\})$; b = symeval(BesselK, 2, x);

In this example, we wish to calculate the BesselK of each of the elements of the matrix x. BesselK objects to this, since it requires an algebraic input. symeval permits the use of matrices for Maple functions that require algebraic input.

SYMLIST REFERENCE - MAPLE COMMANDS

Purpose

Creates a symbolic list.

■ Format

 $s = SYMLIST(v);$ $s = SYMLIST$ $(n, sym);$

Inputs

Outputs

s list

Remarks

A list in Maple is an ordered sequence of expressions or symbols. (A vector is a one dimensional array). The symlist command enables the creation of a list, or the conversion of a vector to a list.

Example

```
1. let v = 1 2 3;
    call symput(v, "v");
    call symrun( "vlst = symlist(v);");
2. vlst = {v1, v2, v3};
3. vlst = symlist(3, v);
```
Example 1 shows how a vector can be converted to a list. Example 2 shows how a list can be created from the individual entities. The same operation is carried out in example 3; this format is useful if there are a large number of elements in vlst.

See also \blacksquare

SYMSET

$\begin{minipage}{.4\linewidth} \begin{tabular}{l} \bf \textit{REFERENCE -} \end{tabular} \end{minipage}$

Purpose

Creates a symbolic matrix.

Format

 $s = SYMMAT$ $(n, m, vlst);$

Inputs

Outputs

s nxm matrix

Remarks

The symmat command creates a symbolic matrix from a list.

Example

```
1. \text{veclst} = \{a,b,c,d,e,f\};m = symmat(2,3,veclst);
2. m = symmat(2,3,{a,b,c,d,e,f});
3. v1 = symvec(\{a, d\});
    v2 = symvec(\{b,e\});
    v3 = symvec({c, f});m = v1^{v}v2^{v}v3;
```
All three examples are equivalent. m is the 2x3 matrix:

 $\begin{array}{ccc} \texttt{a} & \texttt{b} & \texttt{c} \\ \texttt{d} & \texttt{e} & \texttt{f} \end{array}$ d e f

SYMSET REFERENCE - MAPLE COMMANDS

Purpose

Assign symbolic names to a vector.

■ Format

 $slst = SYMSET$ (*v, sn, ord*);

Inputs

Outputs

slst list

Remarks

The symset command creates a symbolic list and assigns each symbol to the corresponding element of a $GAUSS$ vector. v is either the name of an argument that is passed to the procedure, or is the name of a global GAUSS vector or matrix.

Example

```
1. slist = symset(bpar,b,4);
    indx = x1*b1+x2*b2+x3*b3;signa = b4;...
    llfg = gradp(llf, slist);2. b1 = bpar[1];b2 = bpar[2];b3 = bpar[3];signa = bpar[4];slist = \{b1, b2, b3, sigma\};3. vlist = symset(bpar, v, 7:9);
```
REFERENCE - MAPLE COMMANDS SYMSET

The first example is taken from a Tobit estimation, in which the first three elements of the parameter vector, bpar, are the structural coefficients, and the last parameter is the variance. The symset command creates a list of the four parameters in slist, as well as associating b1 through b4 with the respective elements of bpar. slist is needed as an argument to gradp. Equivalent code is shown in example 2. Example 3 shows how a subset of a parameter vector can be associated - in this case, v1, v2, and v3 are associated with bpar[7], bpar[8] and bpar[9] respectively.

SYMVEC REFERENCE - MAPLE COMMANDS

Purpose

Creates a symbolic vector.

Format

 $s = SYMVEC$ (*vlst*);

Inputs

 $vlst$ literal, symbolic list

- **Outputs**
	- s nx1 vector

Remarks \blacksquare

The symvec command creates a vector from a list. In the context of Symbolic Tools , a vector is an nx1 matrix.

Example

- 1. veclst = $\{x,y,z\};$ v = symvec(veclst); 2. $v = symvec({x,y,z})$;
- 3. $v =$ symmat(3,1,{x,y,z});

All three examples are equivalent.

Chapter 8

GAUSS functions

The following list provides information on the functionality of each GAUSS command in a Symbolic Tools context. Note that these functions are case sensitive.

Notes

- 1. In symproc mode, matrix arguments must be passed either as a single parameter, (a scalar symbol), or as a numeric matrix.
- 2. Does not work in symproc mode, since code optimization is not compatible with user specified order.
- 3. Requires numeric argument.
- 4. In symproc mode, requires integer argument.
- 5. Only operates in symproc mode. Note that matrix arguments must be passed as a single parameter, ie. either a scalar symbol, or the name of an existing GAUSS matrix.

Index

AD debugging 43

commands

- summary 25
- symarg 28
- $-$ symdat 46
- $-$ symdebug 29
- $-$ symeval 47
- $-$ symgauss 30
- $-$ symget 31
- symhelp 32
- symlist 48
- symmaple 33 — symmat 49
- symmode 34
- $-$ symout 35
- $-$ symproc 36
-
- symput 39 $-$ symrun 40
- $-$ symstate 42
- symtest 43

data argument 28 debugging 29

help 32

installation 7

kernel

- execute 30, 33, 40
- initialization 42
- retrieve 31

— send 39

output 35

symbolic — element 20 — evaluation 47 — list 20, 48 — matrix 22, 46 — proc 36 — vector 21 syntax 34