

# Spatial Indices of Residential Segregation

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# Introduction

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- To overcome such limitation, new indices have been proposed that adequately account for the relationships of spatial contiguity among residential locations
- The purpose of this talk is to present a preliminary version of `spseg`, a novel user-written Stata program that computes three indices of this kind: two multigroup spatial clustering indices – the spatial dissimilarity index  $\tilde{D}$  and the spatial information theory index  $\tilde{H}$  – and the spatial exposure index  $\tilde{P}_{kl}^*$

# What is residential segregation

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- For example, suppose the inhabitants of Florence are classified into two social classes: white collars and blue collars. If the majority of white collars live in neighborhoods populated primarily by white collars, while most blue collars reside in neighborhoods whose population consists mainly of blue collars, then we can conclude that Florence is characterized by a certain degree of residential segregation by social class

# Measuring residential segregation

- Measuring residential segregation within a given geographical area entails two basic operations: (a) defining the neighborhoods within which individuals live, and (b) quantifying the extent to which the distribution of the social attribute of interest (e.g., social class) varies across neighborhoods (Reardon and O'Sullivan 2004)



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- The indices of residential segregation most commonly used in the literature focus on the second operation, but completely elude the first one, thus failing to take into account the spatial nature of the phenomenon

# Measuring residential segregation

- Expressly, the traditional indices of residential segregation equate the neighborhoods within which individuals live with the administrative units (e.g., the census tracts) into which the geographical area of interest is divided

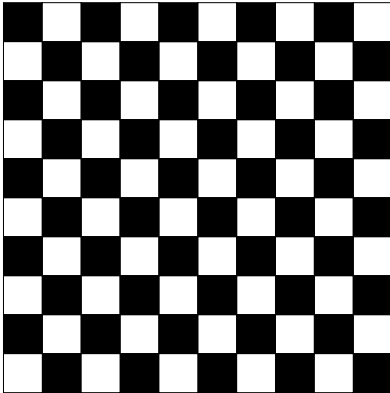
# Measuring residential segregation

- Expressly, the traditional indices of residential segregation equate the neighborhoods within which individuals live with the administrative units (e.g., the census tracts) into which the geographical area of interest is divided
- The establishment of such equivalence between administrative units and neighborhoods, however, raises two issues potentially relevant to the measurement of residential segregation: (a) the checkerboard problem, and (b) the comparability problem

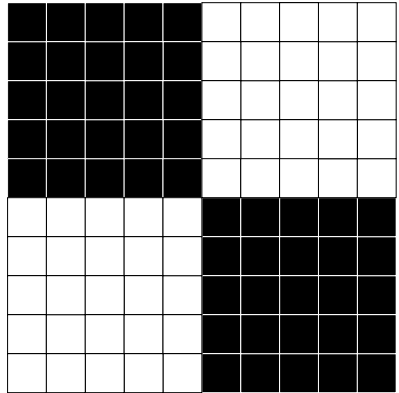
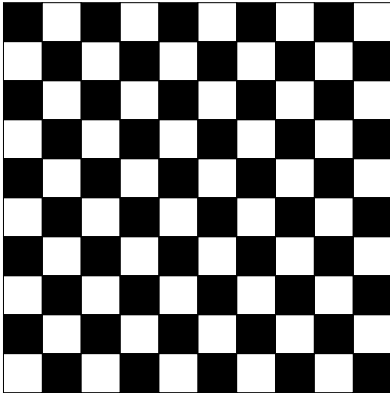
# The checkerboard problem

- The *checkerboard* problem stems from considering each administrative unit in isolation from the others, thus neglecting the overall social composition of its surrounding space

# The checkerboard problem



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- The issue of *comparability* is also very important and comes from the fact that different geographical areas – or the same geographical area at different times – are often divided into administrative units according to different criteria
- This means that, whenever we equate neighborhoods with administrative units and are interested in carrying out a comparative analysis of residential segregation, different areas – or the same area at different times – will likely correspond to different definitions of neighborhoods, thus making any comparison of segregation levels unreliable



## Some solutions

- To tackle the checkerboard and the comparability problems, in the last few years several scholars have devised new indices of residential segregation that take into account the spatial dimension of the phenomenon (e.g., Morgan 1983; White 1983; Morrill 1991; Wong 1993, 1998, 2003; Grannis 2002; Feitosa *et al.* 2004; Reardon e O'Sullivan 2004; O'Sullivan e Wong 2007)

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- These indices are based on definitions of neighborhoods that are less sensitive to the nature of pre-existing administrative units

## Neighborhood definition

- Following Reardon e O'Sullivan (2004), the neighborhood  $N_r$  of each individual  $r$  living in the geographical area of interest is defined as the union of two sets of sub-areas (henceforth *tracts*):

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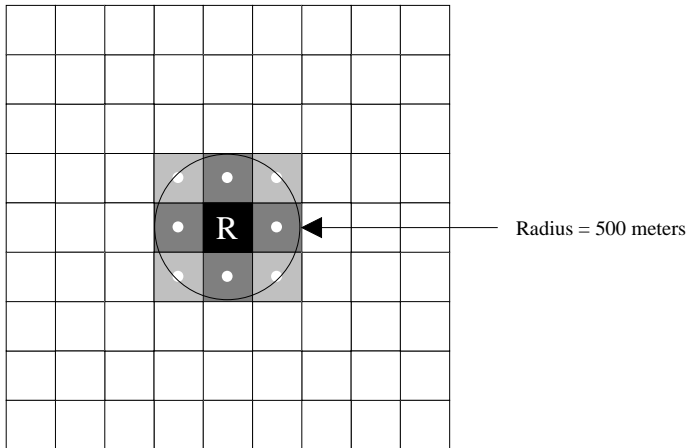
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  - The tract  $R$  where  $r$  lives
  - The tracts whose centroids are located within a given radius from the centroid of tract  $R$
- The social composition of each neighborhood  $N_r$  is then computed by weighting every other individual  $q$  residing in  $N_r$  by a quantity proportional to the inverse of the distance between the centroid of tract  $R$  and the centroid of the tract where  $q$  lives, so as to give greater importance to the nearest neighbors of  $r$ , i.e., to those individuals with whom  $r$  is more likely to interact

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  - the spatial dissimilarity index  $\tilde{D}$
  - the spatial information theory index  $\tilde{H}$
  - the spatial exposure index  $\tilde{P}_{kl}^*$
- The definition of these three indices follows closely that given in Reardon e O'Sullivan (2004)

## Basic quantities (1)

| Symbol   | Description   |
|----------|---|
| $T$      | Number of tracts ( $i, j = 1, \dots, T$ )                         |
| $K$      | Number of groups ( $k = 1, \dots, K$ )                            |
| $n_i$    | Number of subjects living in tract $i$                            |
| $n_k$    | Number of subjects belonging to group $k$                         |
| $n_{ik}$ | Number of subjects living in tract $i$ and belonging to group $k$ |
| $n$      | Total number of subjects living in the study area                 |
| $p_i$    | $n_i/n$   |
| $p_k$    | $n_k/n$   |
| $I$      | $\sum_{k=1}^K p_k(1 - p_k)$                                       |

## Basic quantities (2)

| Symbol            | Description  |
|-------------------|--|
| $\tilde{n}_i$     | $\sum_{j=1}^T w_{ij} n_j$                                      |
| $\tilde{n}_{ik}$  | $\sum_{j=1}^T w_{ij} n_{jk}$                                   |
| $\tilde{p}_{k i}$ | $\tilde{n}_{ik} / \tilde{n}_i$                                 |
| $w_{ij}$          | $f(d_{ij}, r)$   |
| $d_{ij}$          | Euclidean distance between the centroids of tracts $i$ and $j$ |
| $r$               | Radius of the neighborhood                                     |

## Spatial dissimilarity index $\tilde{D}$

The spatial dissimilarity index  $\tilde{D}$  is defined as follows:

$$\tilde{D} = \sum_{i=1}^T p_i \tilde{D}_i$$

where

$$\tilde{D}_i = \sum_{k=1}^K \frac{1}{2I} |\tilde{p}_{k|i} - p_k|$$



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- $\tilde{D}$  can take values ranging from 0 (no segregation) to 1 (maximum possible segregation)

# Spatial information theory index $\tilde{H}$

The spatial information theory index  $\tilde{H}$  is defined as follows:

$$\tilde{H} = \sum_{i=1}^T p_i \left( \frac{E - \tilde{E}_i}{E} \right)$$

where

$$E = - \sum_{k=1}^K (p_k) \log_K(p_k)$$

$$\tilde{E}_i = - \sum_{k=1}^K (\tilde{p}_{k|i}) \log_K(\tilde{p}_{k|i})$$

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- $\tilde{H}$  can take values ranging from 0 to 1, where 0 corresponds to the case in which each neighborhood has the same social composition as the study area (no segregation), and 1 corresponds to the case in which each neighborhood is populated by subjects belonging to one group only (maximum possible segregation)

# Spatial exposure index $\tilde{P}_{kl}^*$

The spatial exposure index  $\tilde{P}_{kl}^*$  is defined as follows:

$$\tilde{P}_{kl}^* = \sum_{i=1}^T \frac{n_{ik}}{n_k} \tilde{p}_{\ell|i}$$

## Spatial exposure index $\tilde{P}_{kl}^*$

- $\tilde{P}_{kl}^*$  expresses the degree of spatial exposure of group  $k$  to group  $l$ , and corresponds to the average proportion of group  $l$  in the neighborhood of each member of group  $k$

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- When  $l \equiv k$ , then  $\tilde{P}_{kk}^*$  expresses the degree of spatial *isolation* of group  $k$ , i.e., the spatial exposure of group  $k$  to itself
- In absence of segregation,  $\tilde{P}_{kl}^*$  corresponds exactly to  $p_l$ , i.e., to the proportion of group  $l$  in the study area
- On the contrary, when there is segregation  $\tilde{P}_{kl}^* \neq p_l$

## The data

- Study area: City of Florence
- Administrative units: Census tracts
- Year: 2005
- Social attribute of interest: citizenship
- Social groups: (1) Italians, (2) Foreigners from “rich” countries, (3) Foreigners from “poor” countries

# Dissimilarity index and Information theory index

Aspatial versions computed by user-written programs `seg` and `spseg` – Selected output

```
. use "K:\Sug09it\Florence-Attributes.dta", clear
```

```
. seg italian forrich forpoor, d h
```

Segregation Measures

| ----- | Dissim. | Inf. Theory |
|-------|---------|-------------|
| Total | 0.2684  | 0.0640      |

```
. spseg italian forrich forpoor, x(xcoord) y(ycoord) band(1) kernel(quartic) d h
```

Spatial indices of segregation

|  | Index              | Estimate |
|--|--------------------|----------|
|  | Information Theory | 0.064    |
|  | Dissimilarity      | 0.268    |

# Dissimilarity index and Information theory index

## Spatial versions computed by spseg – Selected output

```
. spseg italian forrich forpoor, x(xcoord) y(ycoord) band(500) kernel(quartic) d h
Spatial indices of segregation
```

| Index              | Estimate |
|--------------------|----------|
| Information Theory | 0.049    |
| Dissimilarity      | 0.237    |

```
. spseg italian forrich forpoor, x(xcoord) y(ycoord) band(1000) kernel(quartic) d h
Spatial indices of segregation
```

| Index              | Estimate |
|--------------------|----------|
| Information Theory | 0.041    |
| Dissimilarity      | 0.211    |

```
. spseg italian forrich forpoor, x(xcoord) y(ycoord) band(2000) kernel(quartic) d h
Spatial indices of segregation
```

| Index              | Estimate |
|--------------------|----------|
| Information Theory | 0.029    |
| Dissimilarity      | 0.173    |

```
. spseg italian forrich forpoor, x(xcoord) y(ycoord) band(100000) kernel(quartic) d h
Spatial indices of segregation
```

| Index              | Estimate |
|--------------------|----------|
| Information Theory | 0.000    |
| Dissimilarity      | 0.000    |

# Spatial exposure indices

```
. spseg italian forrich forpoor, x(xcoord) y(ycoord) band(500) kernel(quartic) p
```

Spatial indices of segregation

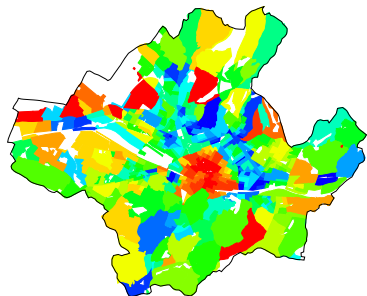
```
Number of tracts =      481
Number of groups =       3
Bandwidth       =     500
Kernel function  =    quartic
```

|                          | Index | Estimate |
|--------------------------|-------|----------|
| Exposure of group 1 to 1 |       | 0.915    |
| Exposure of group 1 to 2 |       | 0.011    |
| Exposure of group 1 to 3 |       | 0.075    |
| Exposure of group 2 to 1 |       | 0.885    |
| Exposure of group 2 to 2 |       | 0.022    |
| Exposure of group 2 to 3 |       | 0.093    |
| Exposure of group 3 to 1 |       | 0.884    |
| Exposure of group 3 to 2 |       | 0.013    |
| Exposure of group 3 to 3 |       | 0.103    |

Marginal distributions

|  | Groups                      | Absolute | Relative |
|--|-----------------------------|----------|----------|
|  | Italians                    | 333668   | 0.912    |
|  | Foreigners - Rich countries | 3988     | 0.011    |
|  | Foreigners - Poor countries | 28262    | 0.077    |

# Spatial dissimilarity index – Local components $\tilde{D}_i$



```
. use "K:\Sug09it\Florence-Attributes.dta", clear

. spseg italian forrich forpoor,      ///
  x(xcoord) y(ycoord) band(500)     ///
  kernel(quartic) d                 ///
  ligen(i) id(id)

. spmap d_i using                    ///
  "Florence-Coordinates.dta",       ///
  id(id) cln(20) fcolor(Rainbow)    ///
  ocolor(none ..) ndocolor(none)   ///
  legend(off)                       ///
  polygon(data("Florence-Outline.dta"))
```

## Conclusion

- `spseg` complements the existing Stata programs for the analysis of segregation, namely `seg` (by Sean F. Reardon), `duncan` (by Ben Jann) and `hutchens` (by Stephen P. Jenkins)



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- `spseg` complements the existing Stata programs for the analysis of segregation, namely `seg` (by Sean F. Reardon), `duncan` (by Ben Jann) and `hutchens` (by Stephen P. Jenkins)
- `spseg` is still at a very preliminary stage and it will take some time before it reaches a publishable form

## References

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