APPLIED GAME THEORY

Nicola Orsini
Karolinska Institutet, Stockholm, Sweden

Debora Rizzuto and **Nicola Nante**
Dept. Public Health, University of Siena, Italy

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GAME THEORY

- Formal study of decision-making where several intelligent and rational players must make choices that potentially affect the interests of the other players.

- It is applied in economy, sociology, biology, health care, and whenever the actions of several agents (individuals, groups or any combination of these) are interdependent.

GAMET

Type of game

- Strategic or Normal Form - Payoff Matrix
- Extensive Form - Game Tree

Solution or equilibrium

- Dominant and dominated strategies (**`domist`**)
- Iterated elimination of strongly dominated strategy (**`elids`**)
- Nash equilibrium in pure strategies (**`neps`**)
- Nash equilibrium in mixed strategies (**`nems`**)
- Maximin criterion (**`maximin`**)
- Backward induction (**`gtree`**)

GENERAL PAYOFF MATRIX

	lab_S2				
lab_S1	clab1	.	clab_c	.	clab_C
rlab1	(#u111; #u211)	.	(#u11_c; #u21_c)	.	(#u11_C; #u21_C)
rlab2	(#u121; #u221)	.	(#u12_c; #u22_c)	.	(#u12_C; #u22_C)
...	...	.		.	
rlab_r	(#u1_r1; #u2_r1)	.	**(#u1_r_c; #u2_r_c)**	.	(#u1_r_C; #u2_r_C)
...	...	.		.	
rlab_R	(#u1_R1; #u2_R1)	.	(#u1_R_C; #u2_R_C)	.	(#u1_R_C; #u2_R_C)

#u1_r_c is the utility for **lab_S1** if lab_S1 chooses strategy r and lab_S2 chooses strategy c

#u2_r_c is the utility for **lab_S2** if lab_S1 chooses strategy r and lab_S2 chooses strategy c

Greatest is the number and greatest is the utility.

DOMINANT AND DOMINATED STRATEGIES

```
. gamet , payoff(2, 2, 0, 1 \ 3, 0 , 1, 1) player1(High
>Low) player2(Buy Not_buy) ls1(Provider) ls2(Customer)
> domist

------------------------------
            |      Customer
 Provider   |    Buy     Not_buy
------------+-----------------
      High  |   (2; 2)    (0; 1)
       Low  |   (3; 0)    (1; 1)
------------------------------

DOMINATED AND DOMINANT STRATEGIES
    Dominated strategies for Provider = High
    No dominated strategies for Customer
    Dominant strategies for Provider = Low
    No dominant strategies for Customer
```

ITERATED ELIMINATION OF STRONGLY DOMINATED STRATEGIES

```
. gamet, payoff(0, 0, 12, 8, 18, 9, 36, 0\ 8, 12, 16, 16, 20, 15, 32, 0 \ 9, 18, 15, 20, 18, 18, 27, 0\0, 36, 0, 32, 0, 27, 0, 0) player1(H M L N) player2(h m l n) ls1(Firm_I) ls2(Firm_II) elids
```

	Firm_II			
Firm_I	h	m	l	**n**
H	(0; 0)	(12; 8)	(18; 9)	**(36; 0)**
M	(8; 12)	(16; 16)	(20; 15)	**(32; 0)**
L	(9; 18)	(15; 20)	(18; 18)	**(27; 0)**
N	**(0; 36)**	**(0; 32)**	**(0; 27)**	**(0; 0)**

Iteration 1

DOMINATED AND DOMINANT STRATEGIES
Dominated strategies for Firm_I = **N**
Dominated strategies for Firm_II = **n**
No dominant strategies for Firm_I
No dominant strategies for Firm_II

	Firm_II		
Firm_I	**h**	m	l
H	**(0; 0)**	**(12; 8)**	**(18; 9)**
M	**(8; 12)**	(16; 16)	(20; 15)
L	**(9; 18)**	(15; 20)	(18; 18)

Iteration 2

DOMINATED AND DOMINANT STRATEGIES
Dominated strategies for Firm_I = **H**
Dominated strategies for Firm_II = **h**
No dominant strategies for Firm_I
No dominant strategies for Firm_II

Firm_I	Firm_II m	Firm_II **l**
M	(16; 16)	**(20; 15)**
L	**(15; 20)**	**(18; 18)**

Iteration 3

DOMINATED AND DOMINANT STRATEGIES
Dominated strategies for Firm_I = L
Dominated strategies for Firm_II = l
Dominant strategies for Firm_I = **M**
Dominant strategies for Firm_II = **m**

Firm_I	Firm_II **m**
M	(16; 16)

NASH EQUILIBRIUM IN PURE STRATEGIES

```
. gamet , payoff(2, 2, 0, 1 \ 3, 0 , 1, 1) player1(High
> Low) player2(Buy Not_buy) ls1(Provider) ls2(Customer)
>  neps

------------------------------
            |      Customer
 Provider   |    Buy      Not_buy
------------+-----------------
      High  |   (2; 2)     (0; 1)
       Low  |   (1; 0)     (1; 1)
------------------------------

NASH EQUILIBRIUM IN PURE STRATEGIES
     1. High Buy     (2; 2)
     1. Low Not_buy (1; 1)
```

NASH EQUILIBRIUM IN MIXED STRATEGIES

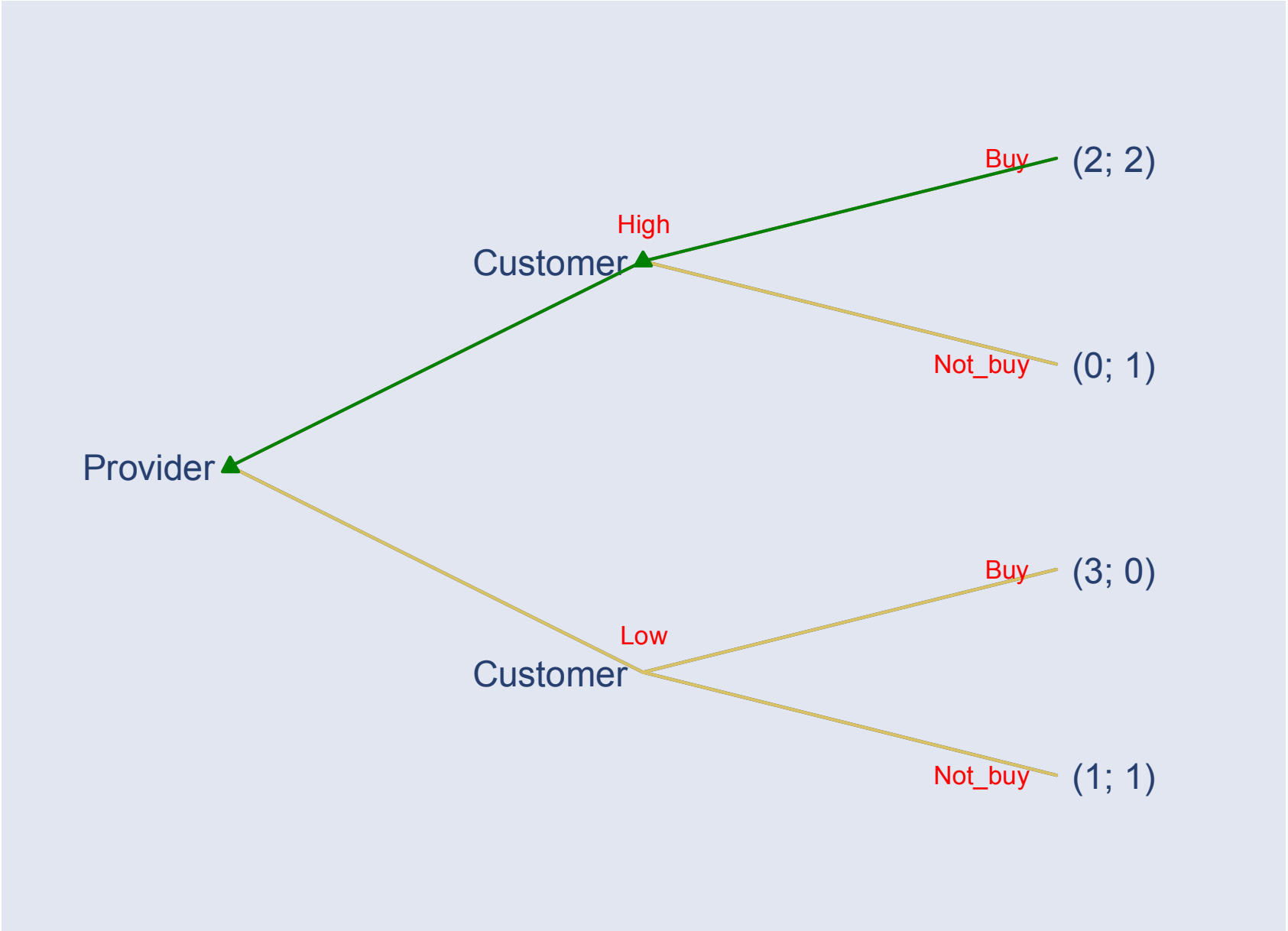
```
.  gamet, pay(0, 0, -10, 10 \ -1, 0, -6, -90)
player1(Not_inspect Inspect) player2(Comply Cheat) ls1(I)
ls2(II) nems

-----------------------------------
            |           II
          I |   Comply     Cheat
------------+----------------------
Not_inspect |    (0; 0)    (-10; 10)
    Inspect |   (-1; 0)    (-6; -90)
-----------------------------------

//    Player I
//    p * 0 + (1-p) * -10 = p * -1 + (1-p) * -6

//    Player II
//    q * 0 + (1-q) * 0 = q * 10 + (1-q) * -90
```

NASH EQUILIBRIUM IN MIXED STRATEGIES

```
    player I
    p = 0.80
    0.80 * Comply + 0.20 * Cheat

    player II
    q = 0.90
    0.90 * Not_inspect + 0.10 * Inspect

    Nash equilibrium in mixed strategy = (-2.00; 0.00)

//    Player I
//    0.8 * 0 + (1-0.8) * -10 = 0.8 * -1 + (1-0.8) * -6 = -2

//    Player II
//    0.9 * 0 + (1-0.9) * 0 = 0.9 * 10 + (1-0.9) * -90 = 0
```

EXTENSIVE FORM GAME

```
. gamet , payoff(2, 2, 0, 1 \ 3, 0 , 1, 1) player1(High
>Low) player2(Buy Not_buy) ls1(I) ls2(II) gtree


------------------------------
           |         II
         I |    Buy     Not_buy
-----------+------------------
      High |   (2; 2)    (0; 1)
       Low |   (3; 0)    (1; 1)
------------------------------

BACKWARD INDUCTION
    Equilibrium path: High Buy
    Payoffs pair: (2; 2)
```


Provider
High
Customer
Buy
(2; 2)
Not_buy
(0; 1)
Low
Customer
Buy
(3; 0)
Not_buy
(1; 1)

```
.  gamet, payoff(0, 0, 12, 8, 18, 9, 36, 0\ 8, 12, 16, 16, 20, 15,
>32, 0 \ 9, 18, 15, 20, 18, 18, 27, 0\0, 36, 0, 32, 0, 27, 0, 0)
>player1(H M L N) player2(h m l n) ls1(Firm_I) ls2(Firm_II) gtree
>title(Extensive form)  // scatter options
```

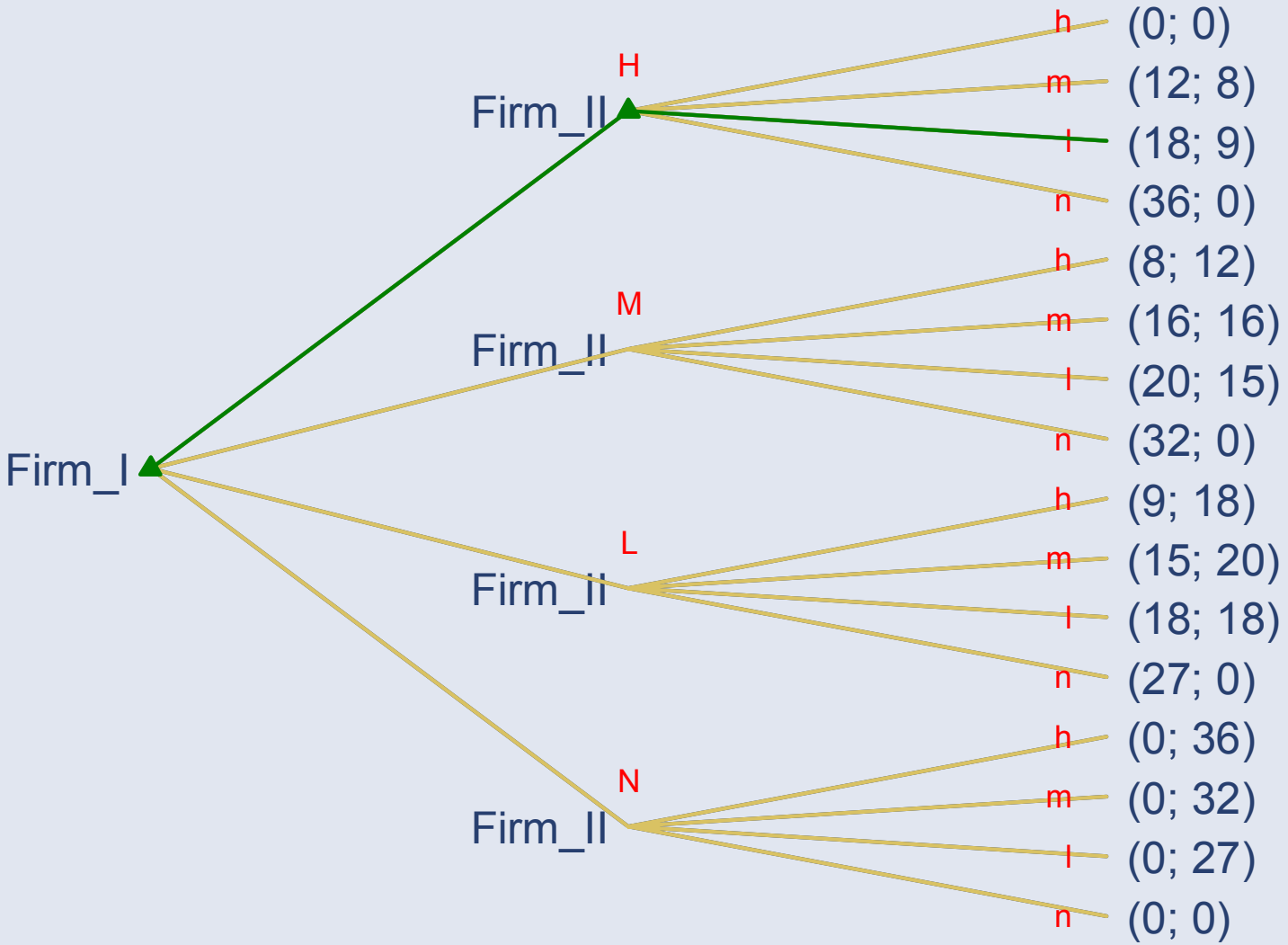
	Firm_II			
Firm_I	h	m	l	n
H	(0; 0)	(12; 8)	(18; 9)	(36; 0)
M	(8; 12)	(16; 16)	(20; 15)	(32; 0)
L	(9; 18)	(15; 20)	(18; 18)	(27; 0)
N	(0; 36)	(0; 32)	(0; 27)	(0; 0)

BACKWARD INDUCTION

Equilibrium path: **H l**

Payoffs pair: **(18; 9)**

Extensive form



ZERO-SUM GAMES

```
. gamet, payoff(-5, 5, 3, -3, 1, -1, 20, -20\5, -5, 5, -5,
> 4, -4, 6, -6\ -4, 4, 6, -6, 0, 0, -5, 5) player1(1 2 3)
> player2(1 2 3 4) maximin
```

	S2			
S1	1	2	**3**	4
1	(-5; **5**)	(3; -3)	(1; -1)	(**20**; -20)
2	(**5**; -5)	(5; -5)	(**4**; **-4**)	(6; -6)
3	(-4; 4)	(**6**; -6)	(0; 0)	(-5; **5**)

```
ZERO-SUM GAME - MAXIMIN CRITERION
    Minimal Column Maximum for S1 = 4
    Maximal Row Minimum for -{S2} = -4
    Saddle-point = 2 3
```

KEYWORDS OF GAMET

- System variable `(_n)` to handle a bi-matrix
- Display payoff table `(tabdisp)`
- Math functions `(max, min)` to seek maximum payoffs
- Lists of elements `(macrolists)` to handle with subscripts
- Create coordinates (y and x) with labels based on players' strategies and overlay scatter plots `(scatter)` to produce a game tree.

ON-LINE MATERIAL

Download **gamet** typing at the Stata command line

```
. net from http://nicolaorsini.altervista.org/stata

. ssc install gamet
```